Roadmap

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Objective

De nition : Game, payo f and Core

De nition

1. A cooperative game (shortly CG) : $(N; \ )$
   N : set of players (actors, economic agents)
   : mapping from P(N) to R satisfying (H) $0.$
2. A payo (allocation) of $(N; \circ)$: x $(x_1; x_2; \ ; x_n)$ P R^N satisfying e ciency, that is, $x(N)$ $\sum_{i \in N} x_i$ (N).
3. A payo x $(x_1; x_2; \ ; x_n)$ is :

Non emptiness of the core of the CG pN; q

De nition : balancedness coe cients

1. A nonempty family of coalitions of N is balanced if there exists a sequence ˝ p $_T$q$_{T\in}$ of positive reals numbers satisfying :
   @i P N; $\sum_{T\in \; ; \; i\in T}$ $_T$ 1:
2. pN; q is balanced if for all balanced family of coalitions of N with balancedness coe cients p $_T$q$_{T\in}$, we have : $\sum_{T\in}$ $_T$ pTq ¤ pNq.

p $_T$q$_{T\in}$: balancedness coe cients of :

NSC for non-emptiness of the core or stability of CG pN; q

Theorem (Bondareva 1963 and Shapley 1967)

A CG pN; q is stable if and only if it is balanced.

Cooperative Game with Multi-cooperation

Definition of SCG and SCGMC

1. A structured cooperative game (shortly SCG) : (N, v, S) where (N, v) is a CG and S is a recovering of N.
2. Cooperative game with multiple cooperations (shortly SCGMC) : The SCG (N, v, S) where
some elements of S are not pairwise disjoints, that is, S is not a partition of N.

Payoff of a Cooperative Game with Multi-cooperation

Let (N, v, S) be a SCG with $S = \{R_1, \dots, R_m\}$.
Payoff of (N, v, S) : a sequence $x = \{x_{i,k}\}_{i \in N,\ 1 \leq k \leq m}$ of real numbers satisfying efficiency on coalitions of the recovering S, that is,

$$\forall k \in \{1, 2, \dots, m\},\ x(R_k) = \sum_{i \in R_k} x_{i,k} = v(R_k). \tag{1}$$

Cooperative Game with Multi-cooperation : 1st main result

Characterization of elements of $C(N, \;, S)$

Let $x = \{x_{i,k}\}_{i\in N,\ 1\leqslant k\leqslant m}$ be a payoff of $(N, \;, S)$ be a SCG with $S = \{R_1, \ldots, R_m\}$.
$x \in C(N, \;, S)$ if and only if

$$\forall k \in \{1, 2, \ldots, m\},\ \forall S \subseteq R_k,\ \sum_{i\in S} x_{i,k} \geq \;(S).$$

Cooperative Game with Multi-cooperation : On the path of stability

De nition

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Insurance market model : Coalitions gains (inspired from Eckardt, 2007)

Gross gain of in exchange between the insurer and the insured a_i and the characteristic function

- Direct cooperation : $V_{a_i}^D \quad C_{a_i}^D \quad (T^D_{insured} \quad T^D_{insurer}).$
- Intermediary exchange : $V_{a_i}^I \quad C_{a_i}^I \quad (T^I_{insured} \quad T^I_{insurer}).$

(K) is given by :

$$\begin{cases} \sum_{a \in K \cap N_a} (V_a^D \quad C_a^D) \quad (T^D_{K\cap N_a} \quad T^D_{ins;K\cap N_a\}}) & \text{if } A \in K; \exists a_i \in K \text{ and } I \notin K; \\ \sum_{a \in K \cap N_a} (V_a^I \quad C_a^I) \quad (T^I_{K\cap N_a} \quad T^I_{ins;K\cap N_a}) & \text{if } A \in K; \exists a_i \in K \text{ and } I \in K; \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

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Insurance market model : second contribution with Game Theory approach

ISGC with intermediary exchange

The ISGC p$N;\ \ ;S$q is in an intermediary exchange if I P $\underset{K\in S}{\text{X}} K:$

Proposition : Fourth main result

Let $(N;\ \ ;S)$ be a ISCG with an intermediary exchange.
The following assertions are equivalent :

1. Condition IE_2 is satis ed and the total transaction cost mapping T is sub-additive.
2. Cp$N;\ \ ;S$q H$:$

T : 2^{N_a} Ñ R$^+$ where @F P 2^{N_a}; TpFq T_F^I $T_{ins;F}^I$.

T is sub-additive, that is,
@$F;G$ P 2^{N_a}; F X G H ùñ TpF Y Gq ¤ TpFq TpGq$:$

Summary

Introduce the rst concepts of a cooperative game with multiple cooperations : recovery of all players, payo , dominance between

Summary

- Introduce the rst concepts of a cooperative game with multiple cooperations : recovery of all players, payo , dominance between payo s, core of the game and game stability.
- Characterize the elements of the classical core of the new game, Establish stability of cooperative game with multiple cooperations.
- Establish the conditions under which it would be in the insurer's interest to draw up each contract with a policyholder (individual contract) through an intermediary.

Open question

Study an insurance market with several insurance companies and several intermediaries.

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