

Optimal Savings and Portfolio Choice with Risky Labor Income and Reference-Dependent Preferences

Servaas van Bilsen
Amsterdam School of Economics
University of Amsterdam
Netspar

based on joint work with

Roger Laeven and Theo E. Nijman

July 26, 2023

1. Introduction

- | How to optimally **save** and **invest total wealth** over the life cycle has been studied extensively.
- | **Human capital** constitutes the largest part of total wealth.
 - | For average US household, it is 90% of total wealth.
- | Impact of **risk-free** and **tradable labor** income on optimal choice is well understood.
 - | Bond-like human wealth diversifies stock return risk.
- | Some authors explore the impact of **non-tradable risky labor** income but assume **traditional** preferences.
 - | CRRA or Epstein-Zin.
- | We explore joint impact of **reference-dependent** preferences and **non-tradable risky labor** income on optimal savings and portfolio decisions.

Risky Labor Income and Reference-Dependent Preferences

- | Labor income is **not risk-less** as has been vividly illustrated by the recent Covid-19 crisis.
 - | US unemployment rates rise from 3.5% to 14.7%.
- | A large experimental and empirical literature has shown **substantial deviations** from **traditional** preferences.
 - | Reference-dependence is one of the strongest empirical phenomena in decision under risk.
- | To understand how **risky human wealth** affects optimal savings and portfolio decisions is of great importance.
- | This paper analyzes this question for an individual with **reference-dependent preferences**.

Three Main Findings

2.

Three Main Findings

3. Optimal investment strategy is **more conservative** compared to the case with risk-less labor income and CRRA preferences.
 - | **Non-tradable risky labor income** causes the optimal share invested in the risky stock to decrease.
 - | An **endogenous reference** level has two additional counteracting effects on the optimal portfolio share.
 - | For a typical range of parameter values, we find the net effect yields a **reduction** in the optimal share invested in the risky stock.

Relating Our Findings to Empirical Analysis

- | Our analysis generates several **testable implications**.
- | We briefly explore how our main findings relate to **real monthly savings data**.
- | Using monthly data on total expenditures and incomes, we test the excess sensitivity of the optimal savings rate and the heterogeneous response of optimal savings rate.
- | Consistent with our main findings, we find **excess sensitivity** of the optimal savings rate; and find that the optimal savings rate of a **low-income** individual exhibits **higher degree** of excess sensitivity than that of a high-income individual.

2. Model

Preferences:

- | Denote by $c(t)$ and $h(t)$ the individual's **consumption level** and **reference level** at time t .
- | Expected **lifetime utility** is given by

$$U = E_0 \int_0^{T_D} e^{-\rho t} u(c(t), h(t)) dt ;$$

with $\rho > 0$ time preference rate and T_D the date of death.

- | We impose weak assumptions on the utility function u .
- | In the base model, we assume that the reference level satisfies

$$dh(t) = (\delta - \rho) h(t) dt ;$$

where the depreciation rate δ is allowed to be time-dependent.

- | In a more general specification, the reference level is allowed to depend not only on **own past consumption** but also on past consumption of the individual's **neighbors** and individual past **labor income**.

Model

State Variables, Individual Labor Income, and Financial Market:

- I We consider an economy with two state variables: non-tradable risky labor income $Y(t)$ and the risky stock price $S(t)$.
- I We assume generic dynamics of individual labor income, driven by a Brownian motion $Z_Y(t)$.
- I We assume the following dynamics for the stock price $S(t)$ and the price of a risk-less asset $B(t)$:

$$\begin{aligned}dS(t) &= (r + \sigma_S \sigma_S) S(t)dt + \sigma_S S(t)dZ_S(t); \\dB(t) &= rB(t)dt;\end{aligned}$$

where $\sigma_S \in \mathbb{R}$ denotes the market price of stock return risk, $\sigma_S > 0$ models the stock return volatility, $Z_S(t)$ is a Brownian motion, and $r \in \mathbb{R}$ denotes the risk-less interest rate.

- I We allow $Z_S(t)$ and $Z_Y(t)$ to be correlated, and denote their correlation coefficient by $\rho_{SY} \in [-1; +1]$.

Dynamic Budget Constraint

- | Denote by $\alpha(t)$ the share of pension wealth $F(t)$ invested in the risky stock at adult age t .
- | The individual's **dynamic budget constraint** is given by

$$dF(t) = (r + \alpha(t) s - s) F(t) dt + \alpha(t) s F(t) dZ_s(t) + (Y(t) - c(t)) dt$$

- | Pension wealth grows because of two reasons:
 - (i) **investment results**;
 - (ii) **new savings** $Y(t) - c(t)$.

Dynamic Optimization Problem

- I The individual faces the following dynamic maximization problem:

$$\max_{c(t); \lambda(t)} E_0 \int_0^{T_D} e^{-\int_0^t \lambda(s) ds} u(c(t), h(t)) dt$$

$$dh(t) = (\lambda(t) - \delta) h(t) dt$$

$$dF(t) = (r + \lambda(t) - \delta) F(t) dt + \lambda(t) F(t) dZ_S(t) + (Y(t) - c(t)) dt:$$

- I Maximize expected **lifetime utility** subject to the **reference level** dynamics and the dynamic **budget constraint**.

4. Main Findings

- | For the illustrations that follow, we rely where possible on parameter values from the existing literature.
- | Our main implications remain **qualitatively unchanged** if we vary the values of the parameters within reasonable limits.

Main Finding III: Conservatism

- I Non-tradable labor income risk and reference-dependent preferences lead to a **conservative** optimal **portfolio** strategy.



Welfare Costs

A strategy in which the savings rate does not respond excessively sensitive to a labor income shock can be quite **costly** in **welfare** terms.

true parameters		and	minimum welfare loss (in %)
0.05	0.1		38.04
0.1	0.2		35.08
0.2	0.3		30.13
0.3	0.4		26.04
0.4	0.5		23.52

Welfare costs are measured in terms of the relative decline in certainty equivalent consumption.

Data

- | We obtain data from the U.S. Bureau of Labor Statistics (Consumer Expenditure Survey).
- | We use **monthly** data on **labor income** and **total expenditures**.
- | Our dataset runs from January 2020 to August 2021 (20 periods).
- | Our dataset includes 15,381 unique individuals.

Heterogenous Response of Expenditures to Income Shocks

- | Regression model:

$$\log c(t) = \log Y(t-1) + \epsilon(t):$$

- | We divide the data into 3 income groups:

- | Low monthly gross incomes
- | Middle monthly gross incomes
- | High monthly gross incomes

- | Coefficient estimates (all statistically significant):

- | $\hat{\alpha}_{Low} = 0.0270$
- | $\hat{\alpha}_{Middle} = 0.1200$
- | $\hat{\alpha}_{High} = 0.2159$

- | Heterogeneous excess sensitivity.
- | CRRA preferences do not predict any of this.

5. Conclusion

- | We have explored the joint impact of **reference-dependent preferences** and **non-tradable labor income risk**.
- | Three key findings:
 1. **Excess sensitivity** of optimal consumption and portfolio share to labor income shocks. **Withdrawing** pension wealth in a wide range of economic scenarios.
 2. Response is **heterogeneous** and heavily varies with ratio of consumption to reference level.
 3. **Conservative** consumption and investment strategies.
- | **Welfare** losses can be as large as 35%.
- | Findings remain intact in the case in which labor income shocks are **not permanent**.
- | Findings are **consistent** with patterns in monthly savings data.
- | To analyze the optimal policies and to determine the shadow price of labor income risk, we have developed a **non-trivial solution** procedure.

Thank you for your attention!