

# 7ci fgY 8YHJ] g

## I b]hg cZ7fYX]h \* \*

#### Gi a a UfmcZh\Y 7ci fgY

 $H\Y'V]ca\YX]WU``Yb[\]bYYf]b[\ `A\ UghYfg`Dfc`YWhU``ck\ g`Wci\ fgYk\ cf\_`A\ UghYfg`gh\ XYbhg`hc`YI\ dYf]YbWY`fYgYUfWk hfU]b]b[\ `Y]h\Yf`k\ ]h\ ]b`h\Y`GWkcc``cf`k\ ]h\ `Wc```UVcfUh]b[\ `]bgh]hi\ h|cbg`gi\ Wk`Ug`\cgd]hU`g`UbX`7G\ C\ "`Dfc`YWg\ UfY`gY`YWhYX`Vmh\Y`ghi\ XYbhi]b`Wcbgi\ `hUh]cb`k\ ]h\ `U`gi\ dYfj\ ]gcf`WcbXi\ Wh]b[\ `fYgYUfWk\ ]b`Ub`UfYU`cZ]bhYfYghhc\ h\Y`ghi\ XYbh'\=Zh\Y`fYgYUfWk\ hcd]WgY`YWhYX`]g`YI\ hYfbU``hc`h\Y`GWkcc`\ b]j\ Yfg]m\xi\Y`ghi\ XYbha\ i\ ghUffUb[\Y\ Zcf`Ub`]bhYfbU`;\ G6a\ 9`Wc!gi\ dYfj\ ]qcf\#JqgYqqcf"$ 

 $\label{eq:hamiltonian} $$HY'6 \le A - \$\&\$'dfc'YWh]g'WcbXi WhYX'cj Yf'\%'hYfa 'UbX'Wcbhf]Vi hYg'\U'ZcZU'ghi XYbhfg'bcfa U'`Z\`'!h]a Y'`cUX'' Dfc'YWhk cf_'i gi U``m']bWi XYg'U'fYj ]Yk 'cZh\Y'`]hYfUhi fYždfY`]a ]bUfmYI dYf]a Ybhgž']XYbh]Z]WUh]cb'UbX 'YUfb]b[ 'cZbYk 'fYgYUfW\'g_]``g'UbX'dfc'YWhd`Ubb]b[ 'Zcf'gi VgYei YbhfYgYUfW\'Ug'dUfhcZ6 <math display="inline">\le A - \$\&\%''$  DYfZcfa UbWY']b'h\]a'Wci fqY']a'[ fUXYX''

#### 7 ci fgY 5]a g

## 7ci fgY @YUfb]b[ Ci hWca Yg

 $52 H f g i WW g g Z ``mW ca d`Yh J g `W c i f g Y z `mc i 'g \ c i `X `V Y `UV `Y `hc .$ 

@YUfb]b[ 'Ci HWca Y	95 GHJ[Y %7ca dYHYbW]Yg
%":8Yj Y`cd`U`XYg][b`cf`U`dfcWYgg`cf`]bj Ygh][UhY`U`\mdch\Yg]g Zc``ck ]b[`]bXi ghfmUbX`dfcZYgg]cbU``Yb[]bYYf]b[`ghUbXUfXg"	D9%"(ž'D9%")
&"`7 f]h]WU``mfYZYWicb`U`gdYWJU`]ghVcXmcZ_bck`YX[Y`fY`UhYX`hch\Y]f`h\Yg]g`hcd]W	D9%"%Ž*D9%"'
' "'5dd`mgWjYbtjZjWUbX'Yb[]bYYf]b[ 'a Yt\cXg'tc'gc`j Y'U'V]ca YX]WU` Yb[]bYYf]b[ 'dfcV`Ya "	D9&"%*D9&" ž'D9&"(ž'D9' "%ž D9' "'
("'5bU'mgY'XUHU'cVYWHjY'mig]b['eiUbHHUHjY'UbX'aUh\YaUHWU'	

# 5ggYgga Ybh

$$\label{eq:hamiliary} \begin{split} \text{H\YfY'UfY'lk c'UggYgga YbhilUg\_g'UWfcgg'} 6 &\in \text{A} - \$\&\$'\text{UbX'} 6 &\in \text{A} - \$\&\$'\text{inc} \text{if a i ghiWca d'YhY'} \\ \text{Ub']bhYf]a 'fYdcfh'' & 6 &\in \text{A} - \$\&\%''\text{mci a i ghiWca d'YhY'U'gW]Ybh]Z]Wa Ubi gWf]dh'' \text{H\Y'Z]bU''[fUXY'\Zcf} \\ 6 &\in \text{A} - \$\&\%'' &\in \text{A} - \$\&\$'' \text{k} ]``'VY'h\Y' \text{k} Y][ \h'X'Uj YfU[Y'cZmci f'UggYgga Ybhg'\Zfca '6 &\in \A - \$\&\$'f\&\$i g'6 &\in \A - \$\&\$''UbX'\&\A - \$\&\$'' \text{UbX'} &\text{C} \end{align*} \end{split}$$

h\Y'dfc\YWia Ubi gWf]drik YfY`XcbY`]b`Wc```UVcfUh]cb`k ]h\z`cf`i bXYfhU\_Yb`Vmzch\Yfa Ya VYfg`cZh\Y`fYgYUfW\ [fci d`cf`VmYl h\fbU``Wc```UVcfUhcfg"`BchY`h\Uhik cf\_`XcbY`Vmch\Yfg`a i ghVY``]a ]h\X'hc`gh\Ydg`h\Uhik ci `X YbUV`Y`h\Y`gh XYbhhc`i bXYfhU\_Y`h\Y]f`dfc\YWb`bchh\Y`dfc\YWhh\Ya gY`j Yg"`9l Ua d`Yg`cZh\]g`a Um`]bWi XY fVi hbch`]a ]h\X'hc\

- gca Y gi f[Yf]Yg VY]b[ i bXYffU\_Yb Vma cfY Yl dYf]YbWX ``UV Wc``YU[i Yg"
- hlggi Y W hi fYg VY]b[ a U]bhU]bYX cf dfc WrggYX Vm UV Ugg]ghUbhg"
- gi fj YmfYgdcbgY cf'dUhjYbhXUhUVUgYg [ YbYfUhYX cf'UbU`mgYX ]b k \c'Y cf'dUfhmVmch\Yfg"
- Uʻgi VgYWgcb cZh\YʻgUa YʻYl dYf]a YbhU``XUhUʻcVhU]bYX Vm`UV Wc``YU[i YgʻZfca 'UʻdfYj]ci gʻgh Xm'
- dfcWrXi fYg'VY]b['ci hgci fWrX'hc'Ub'YI hYfbU''Wca dUbm'
- 8 Yq][ b Y Ya Ybhg Wcbhf]Vi hYX Vmmci f'qi dYfj ]qcf'cf'ch\Yf'qh XYbhg"
- I hj]g]b[ fYgi hg cf a cXY g [ YbYfUhYX VmdfYj ]ci g fYgYUfVX gh XYbhg"

#ia i ghVY`cVj ]ci gʻZfca 'h\]gʻgYWfjcbʻh\Uhh\Y`ghi XYbhgʻWcbhf]Vi hJcbʻ]gʻh\Y`j Ugha Uʻcf]hmcZh\Y`k cf\_XYgWf]VYX`]bʻh\]gʻfYdcfhiʻH\Y`gi dYfj ]gcfʻa i ghg][ bʻh\]gʻʻGYY\_`UXj ]WY`Zfca 'mci f`gi dYfj ]gcfʻ]Zmci 'UfY i bgi fY`UVci h'h\]gʻʻ

#### 7CJ =8!% =a dUWhGhUhYa Ybh

#### 5XX]h]cbU`XYhJ]`g

H\YfY`]g`U`XYfU]`YX`XYgWf]dh]cb`cZh\]g`UggYgga YbhhUg\_`cb`h\Y`HYUa g`W\UbbY`"`⊋mci `Xcbfhi\Uj Y`UWWgg hc`HYUa qž'd-

# 5 http://bxubww.F.Yei ]fYa Ybhg

 $\label{thm:conditional} $$H\YfY^{g'bc'c}Z_W^U^*WUgg'ha Y'Zcf'h\g'WcifgY'''Mcif^2UW'ha Y'bYYXg'hc'VY'cf[Ub]gYX'k]h\'mcif gidYfj]gcfz'Ug'mci'UfY'YIdYWfYX'hc'a YYhh\'Ya 'Uh'YUghcbW'dYf'k YY_'''Mci'a ighghj``YbgifY'mcif' Ybfc`a Ybh'UbX'fY[]ghfUh]cb']g'id'hc'XUhY''$ 

# 7ci fgY'GWXYXi `Y

#### J]Yk 'WUgg'h]a YHUV'Y

## H]a YHJV`Y

8 UhY	Hmd Y	7cbhYbh
Gh XmK YY^&% Bcj Ya VYf'!^&( Bcj Ya VYf	5ggYgga Ybh	⇒bhYf]a 'F YdcfhXi Y'A cbXUmK YY_'‱'Uh ‰) - da "'

# FYgci fWyg

#### DfYgWf]VYX F Ygci fWYg

F Ygci fWgk]``VYa UXYUj U]`UVYhc`\Y`d`gh XYbhg'[i]XYh\Ya ]b'h\Y]f'ci fbYmZcf'H\Yg]g'5"

#### 91 hybglcbg

 $\label{thm:condition} $$Mci 'Wbb'Udd`m'Zcf'gdYWJU`'Wcbg]XYfUhJcb'k \Yb']``bYgg'cf'ch\Yf'WJfW a ghUbWfg']bhYfZYfY'k ]h\'mci f UggYgga Ybh'dYfZcfa UbW'''$ 

Ch\Yf'Udd']\Wh\cbg'Zcf'YI h\bg]cb'cZgi Va ]gg]cb'cZh\Yg]g'f\Ydcfhg'f\Y"[ "'Yei ]da Ybh\Vf\U\_Xck bž\Yh\\'L

- %" 8 ]qW qq h\Y dcqq]V]`]micZUb YI hYbq]cb k ]h\ mci f'qi dYfj ]qcf'Zfqh"
- &" F Yei Yghg WUb h\Yb VY ``cX[ YX Vmh\Y gh XYbh\\YfY <u>\hd.##]bmi f"Wca #mr&ndmj</u>" H\Y gi dYfj ]gcf'k ]`` h\Yb fYW]j Y Ub Ya U]`Ug\_|b[ h\Ya hc Uddfcj YžVYZcfY ]h]g YgWU`UhYX hc h\Y XYW]g]cb dUbY`"
- ' "FYeiYghaighVY`cX[YXVmKYY\_\*\*cZhYfa"
- ("DUbY``XYW]g]cb'k]``'VY'a UXY'VmYbX'cZkYY\_'+"
- ) " H\Y`XYV][g]cb`k]``VY`a UXY`VmU`dUbY``—`Wcbg]gh]b[ `cZh\Y`<cG`fcf`h\Y]f`bca ]bYYŁž'H\Yg]g 7 ccfX]bUhcfž'UbX`%ch\Yf`dYfgcb"
- \*" Ghi XYbhgʻg\ci `X'VY`U`YfhYX'hc`h\Y`ZUWhh\Uhih\]gʻ]gʻbch[i UfUbhYYXžUbX'h\i gʻg\ci `X'bchfY`mcb [Yhh]b['Ub`YI hYbg]cb"
- +" Hmd]\WU``mžYI hYbg]cbg`UfY`[fUbhYX`I D`HC`' kYY\_g"`H\Y``Yb[h\cZh\Y`YI hYbg]cb`bYYXg`hc`VYfYei YghYX`UbX`^igh]\ZYX`Vmh\Y`gi dYfj]gcf"`DUbY``k]```XYW]XY`h\Y``Yb[h\cZh]aY`[fUbhYX"

±bXi ghfmVUgYX dfc YWfg

• 8 ]gUV]`]hmGi ddcfhGYfj ]VWg. \https://ddg.##ghi XYbh'i bgk "YXi "Ui #X]gUV]`]hmgYfj ]VWg

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

@UVcfUlrcfmfYdcflrg\*UbX\*a U\*cf\*Ugg][ ba Yblrg\*k ]\*\*\*fYei ]fY\*U\*Bcb\*D\*U[ ]Uf]ga \*8YWUfUlr]cb\*7cj Yf\*G\YYlr"

5gg][ ba Ybhgʻg\ci `X`VYʻgi Va ]hhYXʻcbʻhja Y"`5`XU]`mdYbU`hmcZ) ı `cZh\Y`a Uf\_gʻUj U]`UV`YʻZcfʻh\Uh Ugg][ ba Ybhk ]```Udd`m'Zcfʻk cf\_ fYVW]j YX`UZhYfʻh\Y`Xi Y`XUhY"`5bmUgg][ ba Ybha cfYʻh\Ubʻ) `XUmg``UhYʻk ]``bch VY`UWWdhYX"'H\Y`cb`mYl Ya dhjcbʻk ]```VYʻk \Ybʻdf]cfʻdYfa ]gg]cbʻZcf``UhYʻgi Va ]gg]cbʻ\UgʻVYYbʻ[ fUbhYX`Vm h\Y`7 ci fgY`WccfX]bUhcf"'9l hYbg]cbgʻk ]```VY`[ fUbhYX`cb`mcbʻa YX]WU``cf`Wca dUgg]cbUhYʻ[ fci bXgʻi bXYf Yl hYa Y`W]fW a ghUbWfg"

## 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

#### D@5; -5F - €A

 $\pm i]g'Yl \ dYWMYX'h\ Uh'U```ghi \ XYbhg'a i gh'Uh'U```h]a \ Yg'gi \ Va \ ]hh\ Y]f'ck b'k \ cf_'Zcf'UggYgga \ Ybh''Gi \ Va \ ]hh\ ci h'WYUf'mUW_bck `YX[ ]b[ 'h\ Y'gci fW''cZVcffck YX'a UhYf]U``cf']XYUg']g \ d`U[ ]Uf]ga "$ 

5```UggYgga Ybhg`k\]W\`mci `\UbX`]b`a i gh\Uj Y`U`<u>B cb`D`U[]Uf]ga '8YWUfUhjcb`7 cj Yf`G\YYh</u>''`H\]g`]g`Zcf Vch\`]bX]j ]Xi U``UbX`[ fci d`k cf\_"'5htUW\`]hhc`mci f`Ugg][ ba Ybh\VYZcfY`gi Va ]hhb[ ']hhc`h\Y`7 ci fgY 7 ccfX]bUhcf`cf`Uhh\Y`GW\cc``C Z]W\''

D`U[]Uf]ga `]g`h\Y`i gY`cZUbch\Yf`dYfgcb'g`k cf\_`cf`]XYUg`Ug`]Zh\Ymk YfY`mci f`ck b"`K \Yb`]h]g`bYWfggUfmcf XYg]fUV`Y`hc`i gY`ch\Yf`dYcd`Y'g`a UhYf]U``mci `g\ci `X`UXYei UhY`mUW\_bck `YX[Y`k \cgY`k cfXg`cf`]XYUg`h\Ym UfY`UbX`k \YfY`mci `Zci bX`h\Ya `f[]j ]b[ `h\Y`Wca d`YhY`fYZYfYbWY`XYhU]`gž`]bWi X]b[ `dU[Y`bi a VYffg\te"H\Y @YUfb]b[ `7 YbhfY`dfcj ]XYg`Z fh\Yf`]bZcfa Uh]cb`cb`k \UhWcbgh]hi hYg`D`U[]Uf]ga `Uh \hdg.##qh XYbh'i bgk "YXi "Ui #d`U[]Uf]ga

## 5 WUXYa ]W = DZcfa Uhjcb

#### 7CI FG9'9J5@I 5H€B'5B8'89J9@CDA 9BH

Gh XYbhZYXVUW\_`\Ug`\Y`dYX'hc`g\UdY`UbX`XYjY`cd`h\]g`Wci fgYž]bWi X]b[ 'ZYYXVUW\_`cVhJ]bYX`Zfca 'cb! `]bY`Yj U`i Uh]cbg`Ug`dUfhcZI BGK 'g`Ug`dUfhcZI BGK 'g`a m9I dYf]YbWY`dfcWYgg"'Mci 'UfY`\][ \`m YbWci fU[ YX`hc`Wca d`YhY`gi W.`Ub`cb!`]bY`Yj U`i Uh]cb`hck UfX`h\Y`YbX`cZHYfa "': YYXVUW\_`UbX`gi [ [ Ygh]cbg dfcj ]XYX`k ]```VY`]a dcfhUbh`]b`]a dfcj ]b[ 'h\Y`Wci fgY`Zcf`Z h fY`gh XYbhg"

•

#### 85H9G"HC"BCH9

.

#### 57589A = 758J = 79

: cf ]bZcfa Uh]cb UVci h

- "Bchyg'cb'Uggygga Ybhg'UbX'd'U[]Uf]ga ž
- · · · · GdYW/U` · 7 cbg/XYfUh/cbgž
- "GWXcc"Gh XYbh9h\]Wa'CZZWYfž'UbX
- .... 69GG

.

#### Gi dd Ya YbhUfm9l Ua ]bUh]cbg.

Gidd`Ya YblUfm91Ua ]bUh]cbg`Zcf`HYfa '%'&\$&&k ]``VY`\Y`X`cb`fH67Eg\ci`X`mci 'VY`fYei ]fYX`hc`g]hcbY"

 $H\g`Wci\ fgY`ci\ h]bY`gYhg`ci\ hXYgWf]dh]cb`cZWUggYg`Uhh\Y`XUhY`h\Y`7ci\ fgY`Ci\ h]bY`]g`di\ V`]g\YX"`H\Y bUhi\ fY`cZWUggYg`a\ UmW\Ub[Y`Xi\ f]b[`h\Y`HYfa`UZhYf`h\Y`7ci\ fgY`Ci\ h]bY`]g`di\ V`]g\YX"``A\ ccX`Y`g\ci`X`VY Wcbgi`hYX`Zcf`h\Y`i\d`hc`XUhY`WUgg`XYgWf]dh]cbg"``=Zh\YfY`]g`Ubm]bWcbg]ghYbWn]b`h\Y`XYgWf]dh]cb`cZ$ 

? Ybg]b[ hcb 'WUa di g ']g '`c WUhYX"

# 5ddYbX]I . '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±bHYbXYX'@YUfb]b['Ci HWca Yg	
?bck `YX[ Y`UbX`g_]```VUgY	
D9%%7ca dfY\Ybg]j Yžfh\YcfmVUgYX`i bXYfgfUbX]b[ `cZfh\Y`i bXYfd]bb]b[ `bUfi fU``UbX d\mg]WU``gW]YbW'g`UbX`fh\Y`Yb[ ]bYYf]b[ `Z bXUa YbfU`g`Udd`]WUV`Y`fc`fh\Y`Yb[ ]bYYf]b[ `X]gW]d`]bY	
D9%"&"7cbWYdh U``i bXYfgHJbX]b['cZh\Y'a Uh\Ya Uh]Wgzbi a Yf]WJ``UbU`mg]gzigHJh]gh]Wgz'UbX Wca di hYf'UbX']bZcfa Uh]cb'gW]YbWYg'k\]W\'i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" =b!XYdh\i bXYfgHJbX]b[ cZgdYV]JU`]ghVcX]Yg`cZ_bck `YX[ Y`k ]h\]b`h\Y`Yb[ ]bYYf]b[ X]gVJd`]bY	
D9%"( '8]gWYfba YbhcZ_bck `YX[ Y`XYj Y`cda YbhUbX`fYgYUfW\`X]fYWf]cbg`k ]h\]b`h\Y Yb[ ]bYYf]b[ 'X]gWJd`]bY	
D9%) '? bck `YX[Y'cZYb[]bYYf]b[ 'XYg][b'dfUWf]WY'UbX'WcbhYl hi U``ZUWcfg']a dUWf]b[ 'h\YYb[]bYYf]b[ 'X]gW]d`]bY	
D9%"* `I bXYfgHJbX]b[ `cZh\Y`gWcdYž'df]bW]d`Ygž'bcfa gž'UWVci bHJV]`]h]Yg`UbX`Vci bXg`cZ gi gHJ]bUV`Y`Yb[ ]bYYf]b[ `dfUWf]WY`]b`h\Y`gdYW]Z]WX]gW]d`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUh]cb`cZYghUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j ]b[	
D9&"&": `i Ybh'Udd`]WUh]cb'cZYb[]bYYf]b['hYVXb]ei Ygž'hcc`g'UbX'fYgci fWYg	
D9&"' '5dd`]WUh]cb'cZgmghYa Uh]WYb[]bYYf]b['gmbh\Yg]g'UbX'XYg][b'dfcWYggYg	
D9&"('5dd`]WUh]cb'cZgmghYa Uh]WUddfcUWXYg'hc'hXY'WcbXi Wh'UbX'a UbU[Ya YbhcZYb[]bYYf]b[dfcYWng	
DfcZYgg]cbU``UbX`dYfgcbU``Ultf]Vi lYg	
D9' "%"9h\]WU``W&bXi Wh'UbX'dfcZYgg]cbU``UWW&i bHUV]`]hm	
D9' "&"9ZZYWNjjY"cfU``'UbX"k f]MnYb"Wcaaib]WUNjcb"]b"dfcZYgg]cbU``'UbX``UmXcaU]bg	
D9' "' '7 fYUhjj Yž']bbcj Uhjj Y'UbX'dfc!UWhj Y'XYa YUbci f	
D9' "( 'DfcZYgg]cbU`i gY'UbX'a UbU[ Ya YbhcZ]bZcfa Uh]cb	
D9' ") 'C fXYf`ma UbU[ Ya YbhcZgY`ZïUbX'dfcZYgg]cbU``WcbXi Wh	
D9' "* '9ZZYW¶j Y'HYUa 'a Ya VYfg\]d'UbX'HYUa ''YUXYfg\]d	