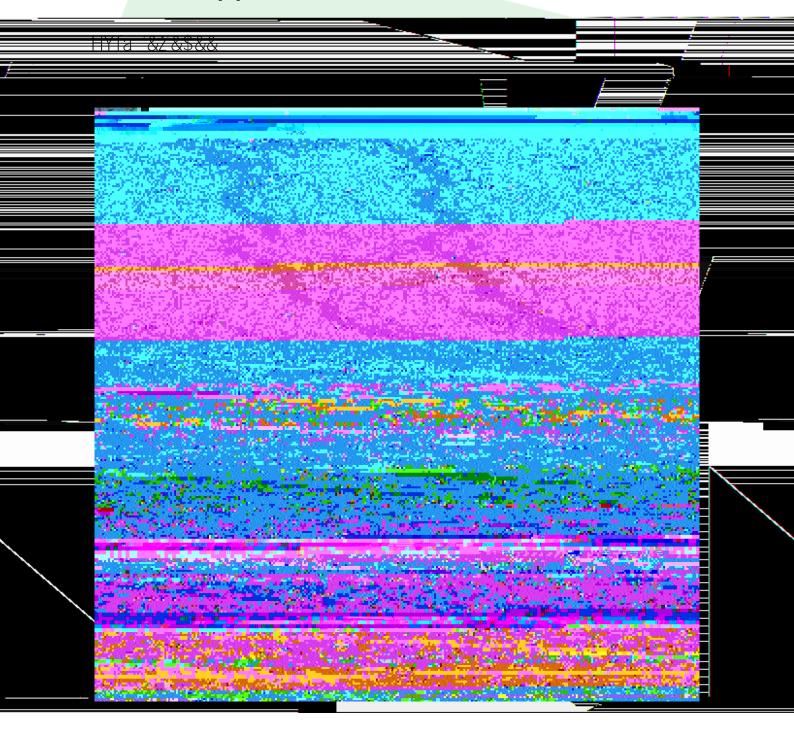


CVEN9702

Dfc YWiD`Ubb]b['UbX'7 cbffc`



7ci fgY^{*}Cj Yfj]Yk

GHJZZ:7cbHJWhi8YHJ]`g

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
GhYj Yb 8Uj]g	g"XUj]g4 i bgk "YXi "Ui	7 cbHJWhj]U A ccX`Y`Zcfi a	F cca '&\$, 6`X['<&\$	Ž*%'f&Ł -',)')\$)&

GWXcc``7cbHJWhʻ±bZcfa Uh]cb

/cifgy 8ynujg
lb]hg~cZ7fYX]h*
Gi a a UfmcZ'h\Y'7ci fgY

 $\label{thm:linear_problem} \mbox{H\Y'd'Ubb]b['dfcWfgg/h]a Y'Ygh]a Uh]b['h\Y'`]b_'VYhk YYb'd'Ubb]b['UbX'Wcblfc'/Wcblfc''gmghYa g/h\Y'']b_'VYhk YYb'd'Ubb]b['UbX'Wcblfc'', where the second of the sec$

UgdYWg`cZh\Y`Wci fgY`UbX`[]j Y`mci `ZYYXVUW_`cb`mci f`dfc[fYgg" GYj YfU``A ccX`Y`Zcfi a g`UfY`Uj U]`UV`Y`Zcf`mci `hc`Ug_`ei Ygh]cbg`UVci h`YWi fY`UbX`k cf_g\cd`a UhYf]U`]b`[YbYfU`ž'UbX`U`gc`UVci hdUfh]W``Uf`Ugg][ba Ybhg"

5XX]h]cbU`7ci fgY +bZcfa Uh]cb

 $\label{thm:condition} $$HY`Y`YWi fY`bchYg`UfY`ZJ]f`mg]a\ d`Y`UbX`UfY`df]a\ Uf]`mdfcj\]XYX`gc`h\Uhighi XYbhg`\Uj\Y`Ub`ci\ h`]bY`hc UbbchUhY`Xi f]b[`h\Y``YWi fY''5g`gi W\`h\YmXc`bchWcbhU]b`h\Y`ghcf]Yg`UbX`UbYWXchYg`h\UhiUfY`i gYX hc`]`i ghfUhY`h\Y`h\YcfmUbX`h\Y`bi a\Yf]WU``YI Ua\d`Yg`h\Uhi=k\cf_'h\fci [\`cb`h\Y`gWfYYb''`H\i\g`]Zmci `a\]gg h\Y``]j\Y``YWi fY`h\Yb`mci `UfY`YI\dYWhX`hc`k\UhW\`h\Y``YWi fY`fYWcfX]b["$

5 ggYgga Ybh :]bU'; fUXY. FZWX' S^YGVWXadfZ[e UagdeWi [** ad_ S*k TWTSeVW a` fZWeg_ aXfZWeUadve Xta_ WSUZ aXfZW SeeVee_ W ffSe] ez: ai WhWd `af S*fab[Ue i [**TWSeeWeeW Tk fZWa` *{ Wcg[i | Wei i Z[WS**fab[Ue _ Sk

H\Y'ei]nnYg'k]``'VY'UXa]b]ghYfYX'h\fci[\'AccX`Y"

H\Y`k\YV`VUgYX`]bhYfZJVW`Zcf`h\Y`ei]nnYg`k]```VY`XYa cbghfUhYX`Xi f]b[`h\Y`gYWcbX``YWn fY`cZh\Y`Zffh\Y`Zffh\Y`ZffdUfh]W\`Uf`ei Ygh]cbg`U`XYa cbghfUh]cb`k]``
vY

[]j Yb'Xi f]b['th\Y'YWi fY'th\Uhth\Y'ei]n']g'fY'YUgYX'UbX'th\Y'UggYgga YbhcZth\Uhei Ygt]cb'k]``'VY'YI d`U]bYX" H\Y'k Y][\t]b['Zcf'YUW\'ei]n'k]``'VY'dfcdcft]cbU'`tc'th\Y'bi a VYf'cZdc]btg'Zcf'th\Y'ei]n'X]gd`UmYX']b A ccX'Y"'H\Ymk]``'bch'VY'U``'th\Y'gUa Y'] U`i Y"

8 YHJ]`YX`ZYYXVUW_`Zcf`YUW\`ei Ygh]cb`k]```VY`dfcj]XYX`h\fci [\ 'h\Y`gUa Y`k YV`VUgYX`]bhYfZJWY`cbY k YY_`UZhYf`h\Y`ei]n`]g`Xi Y"

8i Y'8UhYg'UbX'@UhY'DYbU'h]Yg

; YbYfU`mh\Y'ei]nnYg'k]``VY'XiY'fk c'kYY_g'UZhYf'h\Y'fY`Yj Ubha UhYf]U``\Ug'VYYb`WcjYfYX']b'h\Y WUgg"'H\Y'UWf U``kYY_'h\UhYUW\cZh\Y'ei]nnYg']g'fY`YUgYX'UbX']g'XiY'Wb'VY'ZcibX']b'h\Y'7 cifgY Dfc[fUa"'5``cb`]bY'ei]nnYg'k]``VY'XiY'Uh' da 'cb'h\Y'H\ifgXUm']b'h\Y'kYY_'g\ckb']b'h\Y'7 cifgY Dfc[fUa"

H\YfY`]g`bc`h]a Y``]a]hch\Yf`h\Ub`h\Y`Xi Y`XUhY#]a Yžmci `WUb`df]bhci hh\Y`ei Ygh]cbg`cbY`XUm\UbX`hmdY h\Y`Ubqk Yfq`]bhc`h\Y`Wca di hYf`cb`Ubch\Yf`XUm'

Zmci 'bYYX'hc'gi Va]himci f'ei]n``UhY'h\Yb'hmdY'mci f'Ubgk Yfg']bhc'h\Y''@UhY'Gi Va]gg]cbg'cZ
5gg][ba Ybhg''A ccX`Y': cfi a "'8c'bchi gY'UhhUW\a Ybhg'i b`Ygg'U'ei Ygh]cb'Ug_g'Zcf'U'd]Wh fY'''Mci 'k]```VY dYbU`]gYX') ı

dYf`XUm`UhY`cf`dUfhih\YfYcZVUgYX`cb`h\Y`h]a Y`cZdcgh]b["`Bc`gi Va]gg]cbg`k]``VY`UVWYdhYX`a cfY`h\Ub)`XUmq`f%\$`\ci fgŁ``UhY"

5qqYqqa Ybh&:]bU`9l Ua

GHUFhXUhY. '8 i f]b['Y| Ua 'dYf]cX 5ggYgga Ybh`Yb[h\. '&'\ci fg'Ž'%\$ 'a]bi hYg'fYUX]b['h]a Y Gi Va]gg]cb'bchYg. 'H\Y'Y| Ua 'k]``'VY'UXa]b]ghYfYX'h\fci [\ 'A ccX'Y

Mci f'UV]`]lmhc'i bXYfghUbX'UbX'Udd`mh\]g'h\Ycfmž'Ug'k Y``'Ug'h\Y'ei Ubh]hUh]j Y'hYW\b]ei Ygž'k]```VY'UggYggYX]b'U'WcgYX'Vcc_'Yl Ua ž'k \]W\'k]```hU_Y'&'\ci fg'Xi f]b['h\Y'Zcfa U``Yl Ua 'dYf]cX"'5bmhcd]WWcj YfYX']b WUgg'WUb'VY'Wcj YfYX']b'h\Y'Yl Ua

<i fX`Y`fYei]fYa Ybh

5 'a Uf_'cZUh``YUgh'(\$i `]b'h\Y'Z|bU``Yl Ua]bUh]cb']g'fYei]fYX'VYZcfY'h\Y'WUgg'k cf_']g']bWi XYX']b'h\Y'Z|bU` a Uf_"

K YY_', .'% '>i `m! '&& >i `m	GhcWkUghjWA YhkcXg. DB9H°Ž°A cbhY°7 Ufc Gjai`Uhjcb	
	K YV'E i]n'' 'Xi Y	
	K YV'E i]n'('fY`YUgYX	
K YY_''&) '>i `m!'&- >i `m	A cb]hcf]b['UbX'F Ydcfh]b[
K YY_'%\$.'%'5i [i gh!') 5i [i gh	7 cblfc`ž'7 cblfUWfg	
Ji [i gii	K YV'E i]n'('Xi Y	

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

HYI hVcc

 $\label{thm:compact} $$H\YfY']g'bc'dfYgWf]VYX'hYIhVcc_'Zcf'h\]g'Wci fgY $$GYj YfU``cZh\Y'hcd]Wg'Wcj YfYX']b'h\]g'Wci fgY'UfY'XYhU]'YX']b'a Ubm'hYI hVcc_g'h\Uh'mci 'WUb'Z]bX']b h\Y'`]VfUfm'$

AccX`Y

H\]grigi VYWi\UgrUrA ccXYrg]hY"H\YrUxXfYggr]gr\hhd.##a ccXY"hYrhi bgk "YXi "Ui "

\$hYfUWij Y"Yggcbgrk]``VYrUj U]rUvYrcbrA ccXYrh\Unmci rg\ci `XrWca d'YhYrvYzcfYrh\Yr@YWif fY"

H\Yrg]hYrk]``WcbhU]brUxX]rjcbU`fYgci fWgrZcfmci "H\YrUgg][ba Ybhgrk]``VYr[]j Ybr]brh\YrZcfa rcZcb`]bY

ei]nnYg"8]gW gg]cbrZcfi a gr\Uj YrVYYbrgYhi dr]brA ccXY"E i Ygh]cbgrUvci hih\YrWci fgYzrUgg][ba Ybhgz

YI Ua grYhWUfYrVyghUg_YXrh\fci [\rinkYrX]gW gg]cbrZcfi a gzgcrh\Unu`rgh XYbhgrWbrgYYrh\YrfYd`]Yg"

\$\times UXX]rjcbrcbYrcZh\YrA ccXYYx]gW gg]cbr[fci dgrk]``VYri gYXrUgrUrWUggrYa U]``]gh'r5bmUbbci bWra Ybhgrh\UnwUbbchVYra UXYr]brh\YrYWirfYrk]``VYrdcghYXrhcrh\]gr[fci dzrUbXrk]``VYrZcfk UfXYXrhcrmci fri b]a U]`

UXXfYgg" \frac{1}{2} Jgrl b]j Yfg]hmdc`]Wmh\Uh]bZcfa Uh]cbrgYbhhcrmci fri b]a U]``UXXfYggr\UgrVYYbrfYWf]j YXrVmmci "

\$\frac{1}{2} Jh\YfYZcfYrfYwca a YbXYXrh\Uhmci rWXYW_mci frya U]``fY[i `UfmZcfrUbmUbbci bWra Ybhgrh\Uha][\hbch
VYra UXYr]bryWirfYgrYhW

7 ci fgY 9j U'i Uhcb UbX 8Yj Y'cda Ybh

D'YUgY Waa d'YhY h\Y A m91 dYf]YbWY gh XYbhgi fi YmUhh\Y WabWi g]cb cZh\Y hYfa "

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

D`YUgY`fYZYf`hc`h\Y`A ccX`Y`dU[Y`cZh\Y`Wci fgY`Zcf`Z fh\Yf`[i]XUbW\`cb`UggYgga Ybhigi Va]gg]cb"

I BGK `\Ug`U`ghUbXUfX``UhY`gi Va]gg]cb`dYbU`hmcZ

•)ı 'dYf'XUmž'Zcf'U``'UggYgga Ybhg'k \YfY'U'dYbU'hm'Udd`]Ygž'WUddYX'Uh'Zjj Y'XUmg'f1%&\$'\ci fg\z'UZhYf k\]WX'U'ghi XYbh'WUbbchgi Va]h'Ub'UggYgga Ybhž'UbX'bc'dYfa]hhYX'j Uf]Uh]cb"

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

6Yk UfY "5b" Ugg][ba Ybhh\Uh]bWi XYg"d`U[]Uf]gYX"a UhYf]U``k]```fYWY]j Y"U"\$ı ': U]`žUbX`ghi XYbhg`k \cd"\U[]Uf]gY"a UmZU]``h\Y'Wci fgY"'Ghi XYbhg`k \cd"\U[]Uf]gY"UfY'U`gc"`]UV`Y'hc"X]gW]d`]bUfm'UWf]cbž]bWi X]b[YI Wi g]cb'Zfca 'Ybfc`a Ybh'

 $D`U[]Uf]ga ~]g'hY'i gY'cZUbch\Yf'dYfgcb'g'k cf_'cf']XYUg'Ug']Zh\Ymk YfY'mci f'ck b'''K ~Yb']h]g'bYWYggUfmcf XYg]fUV'Y'hc'i gY'ch\Yf'dYcd'Y'g'a UhYf]U`'mci 'g\ci `X'UXYei UhY`mUW_bck `YX[Y'k \cgY'k cfXg'cf']XYUg'h\Ym$

5ddYbX]I . '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa =bhybXYX=@YUfb]b['Ci hWca Yg	
?bck `YX[Y`UbX'g_]```VUgY	
D9%"%7ca dfY\Ybg]j Yž'h\YcfmVUgYX'i bXYfghUbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]\WJ`'g\WJYb\Wg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbhJ`g'Udd`]\WJV`Y'hc'h\Y'Yb[]bYYf]b['X]g\WJd`]bY	
D9%"&"7cbWYdh U``i bXYfgHJbX]b['cZh\Y'a Uh\Ya Uh]Wgžbi a Yf]WJ``UbU`mg]gžgHJh]gh]Wgž'UbX Wca di hYf`UbX`]bZcfa Uh]cb'gW]YbWYg'k\]W\'i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" '=b!XYdh\'i bXYfghUbX]b['cZgdYW]U`]ghVcX]Yg'cZ_bck `YX[Y'k]h\]b'h\Y'Yb[]bYYf]b[X]gW]d`]bY	
D9%'('8]gWfba YbhcZ_bck `YX[Y 'XYj Y`cda YbhUbX'fYgYUfWX 'X]fYWf]cbg'k]h\]b'h\Y Yb[]bYYf]b['X]gW]d`]bY	
D9%) '? bck 'YX[Y'cZYb[]bYYf]b['XYg][b'dfUWjWY'UbX'WcbhYl hi U`'ZJWcfg']a dUWjb['h\Y Yb[]bYYf]b['X]gWjd`]bY	
D9%"* I bXYfgHJbX]b['cZh\Y'gWcdYž'df]bW]d`Ygž'bcfa gž'UWVci bHJV]`]h]Yg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[]bYYf]b['dfUWfJWY']b'h\Y'gdYW]ZJWX]gW]d`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUh]cb`cZYgHUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j]b[
D9&"&": `i YbhiUdd`]WUh]cb'cZYb[]bYYf]b['hYVXb]ei Ygžhcc`g'UbX'fYgci fWYg	
D9&" '5dd`]WUh]cb`cZgmghYa Uh]WYb[]bYYf]b[`gmbh\Yg]g`UbX`XYg][b`dfcWYggYg	
D9&"('5dd`]WUh]cb'cZgmghYa Uh]WUddfcUWXYg'hc'h\Y'WcbXi WhUbX'a UbU[Ya YbhcZYb[]bYYf]b[dfcYWng	
DfcZYgg]cbU`'UbX'dYfgcbU`'Ultf]Vi hYg	
D9' "%'9h\]WU`'WcbXi Wh'UbX'dfcZYgg]cbU`'UWWci bHJV]`]hm	
D9' "&'9ZYWAj Y'cfU`'UbX'k f]MYb'Wcaaib]WUAjcb']b'dfcZYgg]cbU`'UbX'`Um'XcaU]bg	
D9' "' '7 fYUhjj Yž']bbcj Uhjj Y'UbX'dfc!UWhj Y'XYa YUbci f	
D9' "('DfcZ/gg]cbU`i gY'UbX'a UbU[Ya YbhcZ]bZcfa Uh]cb	
D9' ") 'C fXYf'mia UbU[Ya YbhcZgY`Zi'UbX`dfcZygg]cbU``WcbXi Wh	
D9' "* '9ZZYWNjj Y'NYUa 'a Ya VYfg\]d'UbX'NYUa 'YUXYfg\]d	