

# 7ci fgY<sup>·</sup>Cj Yfj ]Yk

## GhJZZ 7 cbhJWh 8 YhJ] `g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
5bbU <sup>-</sup> 6fi W				

# 7 ci fgY'8YHJ]`g I b]hg'cZ'7fYX]h'\* Gi a a UfmcZ'h\Y'7ci fgY I W{hW[`Sf[\_WaXW WdYk fcS`

	W/[hW[`Sf[_'	WaXW WoYk foS`	e[f[a`žl ZSf	$SdWfZW[_$	b'{USf[a`eaXfW	'JZ`a^aY[US^UZS	S`YWS`VSUMS`
W	'WaYk XgfgaM1 I	[^agdW WdYk	XgfgdWTWUW	fdS1feWW a	dV[efd[TgfVW1	ZSf [e fZWda W	/aXfZWe_SdfYd[V1

@YUfb]b[ 'Ci HWca Y	95 GHJ[Y %7ca dYhYbW]Yg
("'5VWW'gg'a Uf_YhUbX']bXi glfmXUlU'UbX'UbU'mgY'fYbYk UV'Y YbYf[mVi g]bYgg'a cXY'g'UbX'l\Y']a dUWng'cZa Uf_YhfY[i 'Uhcfm UffUb[Ya Ybhg'UbX'dc']Whi]bglfi a Ybhg"	D9%" žD9%") žD9' "' žD9' "( ž D9' "*
) "`7 f]h]WU``mYj U`i UhY`fYbYk UV`Y`YbYf[ mUbX`W]a UhY`dc`]Wn YZYWNj YbYggžYZ]W]YbWnžYei ]lmUbX`ZYUg]V]`]lmž'UbX`dfcdcgY`dc`]Wn UbX`a Uf_YhfYZcfa "	D9%" žD9%"(žD9' "&žD9' "'

HYUW ]b[ 'GhfUhY[ ]Yg'

#### 5ggYgga Ybh

5ggYgga YbhiWcbg]ghgicZhi hcf]U``ZJWj`]hUhjcbž`YWi fYidUfhjWjdUhjcbž'U'k f]hhybiUgg][ ba Ybhifk ]h\i]bhyfa YX]UhY UggYgga YbhiUbX`ZYYXVUW\_ighU[ Ygłž'UbX`U'Z]bU``YI Ua ]bUhjcbidUdYf"'8YhU]`gicZUgg][ ba YbhiUbX'hi hcf]U``ZJWj`]hUhjcbihUg\_gik ]```VYidfcj ]XYX'j ]U'A ccX`Y"

#### 5qqYqqa YbhFUhcbUY

DfYdUfUhjcb Zcf'UbX`ZUWj`]hUhjcb cZhi hcf]U`X]gW gg]cbg'k ]```YbWti fU[Y'ghi XYbhg'hc'UWhj Y`mYb[U[Y']b'h\Y Wti fgY'h\fci [\ci hh\Y'gYa YghYf''K f]hYb'Ugg][ba Ybhg'k]``fYei ]fY'ghi XYbhg'hc'Wt``YWhjbZcfa Uhjcb'UVci h YbYf[mdc']WjYg'UbX'fY[i `Uhjcbgž'h\Y]f'WtbhYl hz'Udd']WUhjcb'UbX`XYhJ]`YX`XYg][b/hc'i gY'a cXY``]b[ž'UbU`mg]g UbX'ZfUa Yk cf\_g'hc'Yj U'i UhY'dc`]WmYZZYWjj YbYggž'YZJWjYbWnUbX']a dUWhcb'X]ZZYfYbhghU\_Y\c`XYf'[fci dg" H\Y'Z]bU``Yl Ua'k]``hYghi bXYfghUbX]b['cZh\Y'WtbW\dhg']bhfcXi WXX']b'h\Y'Wti fgY'UbX'h\Y]f'Udd`]\WUhjcb'hc dc`]WnUbU`mg]g"

Hi hcf]U`g`k]```Zc``ck 'cbY'cZ'hk c'Zcfa Uhg.

#žEfgVW f V V [eUgee[a`S`V!adSUf[h[f]We dWSfWV fa fZWUgddW f fab[Už

- ; fci dgžYUW\`cZUfci bX`(!\*`ghi XYbhgžk]``VY`Zcfa YX`]b`h\Y`Z]fghhi hcf]U`": cf`h\Y`hi hcf]U``hcd]Wg`bi a VYfYX]b`h\Y`Wci fgY`gW\YXi`Y`VY`ck ž[fci dg`k]``VY`U``cWUhYX`hi hcf]U``dfYdUfUh]cb`UbX`ZUW[`]hUh]cb`hUg\_g"
- ; i ]XUbVV`UbX`UggYgga YbhVVf]HYf]U`k ]```VY`dfcj ]XYX`cb`ZUVJ`]HUH]b[ 'h\Y`h hcf]U`''; fci dg`UfY`YbWci fU[ YX`hc Wcbgi `h'h\Y]f`h hcf`Zcf`gi ddcfhicf`ZYYXVUW\_`]b`fY`Uh]cb`hc`h\Y]f`dfYdUfUh]cb"`H\Y`I BGK `@YUfb]b[ '7 YbhfY U`gc`dfcj ]XYg`UXj ]WY`hc`gh XYbhg`cb`dUfh]W]dUh]b[ ']b`h hcf]U``X]gW gg]cbg"

\$žEgbbadfXadSee[Y`\_Wf

H\Y`Ugg][ ba Ybh]gʻghU[ YX`UbXʻZYYXVUW\_ʻ]gʻdfcj ]XYX`hcʻgi ddcfhigh XYbh`YUfb]b[ 'UbXʻg\_]``gʻXYj Y`cda Ybh UbX`]a dfcj Ya YbhcZh\Y`ei U`]hmcZh\Y`Z]bU``gi Va ]gg]cbg"' \dir k YY\_'' \tight zgh XYbhgʻgi Va ]hi\\Y]f`Ugg][ ba Ybhk cf\_`Zcf`ZYYXVUW\_\Zca 'h\Y]f`ligf" bcf"

Mci 'a UmZYY`'th\Uhith\Y'tha Y'fYei ]fYX'tc'Wca d`Yth''th\Y'Hi tcf]U``UbX'5gg][ ba YbtiYl YfWjgYg']g X]gdfcdcfthcbUth'm`Uf[ Y'Wca dUfYX'tc'th\Y]f'k Y][ \thjb[ ']b'th\Y'UggYgga YbtitUV`Y'' < ck Yj Yfž'mci 'g\ci `X'gYY th\YgY'Ug'Zcfa Uthj Y'UggYgga YbtiUbX'bcth'th\UtidYfZcfa ]b[ 'k Y``']b'th\Y'Z]bU`'Yl Ua 'k ]``fY`mcb'\_bck `YX[ Y UbX'g\_]``g'XYj Y`cdYX']b'ti tcf]U'g'UbX'Ugg][ ba Ybtg"

5ggYgga Ybh'tUg_	K Y][ \h	8iY'8UHY	7 ci fgY @YUfb]b[ Ci HWca Yg 5ggYggYX
%":Hi hcf]U`:: UWj]hUhjcb:	&\$ı	B chi5dd`]WUV`Y	%ž'&ž' ž'(
&"`5gg][ ba Ybh	(\$1	: ]bU`fYdcfhXiY'kYY_'%\$ k ]h\f)bhYfaYX]UhY UggYggaYbhg'XiY': f]XUm kYY_'' 'UbX'Xif]b['kYY_'* WUgg"	&ž' ž'(ž')

5ggYgga Ybh'tUg_	K Y][ \h	8iY'8UHY	7 cifgY`@YUfb]b[ Cio <b>'NYN</b> ANSE aY Yojj555og <b>og Yo</b> gogNXX
' "': ]bU`'91 Ua	(\$1	91 Ua "dYf]cXž"XUhY"H67	%ž`&ž`'

5ggYgga Ybh% Hi hcf]U`: UWj`]hUhjcb fl fci dŁ

5ggYgga Ybh`Yb[h...&sa]bi hYg

Gi Van John Strategram Gi Van John Gi Van John Gi Wa John Gi Van John Gi Van John Gi Wa John Gi Wa

 $\label{thm:condition} DfYdUfUhjcb: Zcf: UbX: ZUWj: ]hUhjcb: cZhi. hcf]U::X]gWi.gg] cbgž'k] ```YbWci. fU[Y'ghi. XYbhg'hc'UWhj; Y`mYb[U[Y']b'h\'\% of the condition of the condi$ 

5ggYgga Ybh`Yb[h\.'&'\ci fg 8i Y`XUhY.'9l Ua 'dYf]cXž'XUhY'H67 A Uf\_g'fYh fbYX.''l dcb'fY`YUgY'cZZbU`fYgi `hg

 $\label{thm:limit} $$HYYHUa^{-}]g'U'\&'\cif'k f]hhYb'YHUa^{bUh}cbžk \]WX'hYghg'ibXYfghUbX]b[^{cZh}Y'WcbW'dhg']bhfcXi WYX']b'hXY WcifgY'UbX'hXY]f'Udd`]WhJcb'hc'dc`]WhUbU`mg]g''$ 

#### 5ggYgga Ybh'Wf]hYf]U

E i Ygh]cbg'a UmVY`XfUk b`Zfca `UbmiUgdYWhcZh\Y`Wci fgYž'i b`Ygg'gdYW]ZJWU``m']bX]WUhYX'ch\Yfk ]gY`Vmih\Y`YWhi fYf"

#### 5 hmy b X Ub Wy F Yei ] f Ya Yb hg

Ghi XYbhgʻUfYʻYbWci fU[YX`hcʻUhhYbX`h\Yʻ`YWhi fYgʻ`]j YžhcʻUg\_ʻei Ygh]cbgʻUbX`Yb[U[Yʻ]bʻX]gWi gg]cb" FYWcfX]b[gʻcZ`YWhi fYgʻk]```VY`dfcj ]XYX`UbX'g\ci `X`VY`fYj ]Yk YX`k \YfY`UhhYbXUbWi`]gʻbchdcgg]V`Y"`

DfYdUfUhjcb Zcf'UbX'UthYbXUbW'Uthh hcf]U`g']g'fYei]fYX"5XYeiUthY'Yld`UbUhjcb'k]``bYYX'hc'VY'dfcj]XYX'Zcf a ]ggYX'h hcf]U`g'ch\Yfk]gY'U'aZUM<u>Y</u>cZnYfc'k]``VY'fYWcfXYX'Zcf'h hcf]U``ZJWj]hUhjcb']b'h\Uhk YY\_"'Gh XYbhg k\c'Uff]j Y'j Yfm`UhY'cf'`YUj Y'h\Y'h hcf]U``VYZcfY'h\Y'YbX'cZWUgg'k]h\cihYld`UbUhjcb'k]``'U`gc'fYWY]j Y'U a Uf\_'cZnYfc"

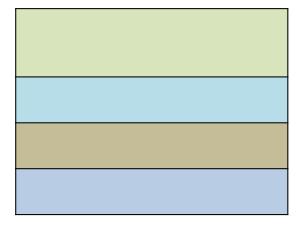
#### 7 ci fgY GWX YXi `Y

 $H\ [g^{-}]g^{-}U^{-*}ib]hcZWYX]hflc7EWcifgY^{-}UbX^{-}]bjc^{-}Yg^{-}(\ \cifg^{-}dYf^{-}kYY_{-}cZZUWY!hc!ZUWY\#cb^{-}]bY^{-}WcbhUWh^{-}$ 

H\Y`bcfa U``k cf\_`cUX`YI dYWfUhjcbg`cZU`ghi XYbh'UfY`UddfcI ]a UhY`m`&) `\ci fg`dYf`hYfa `Zcf`YUW\`I C 7 ž ]bWi X]b[ `WUgg`WcbhUWfi\ci fgž'ch\Yf``YUfb]b[ `UWfjj ]hJYgž'dfYdUfUhjcb`UbX`hJa Y`gdYbh'cb`U```UggYggUV`Y k cf\_"

Mci 'g\ci `X'h\YfYZcfY'U]a 'hc 'gdYbX'UVci h'% -%( `\ci fg'dYf'k YY\_'cb'h\]g'Wci fgY'h\fci [\ci hh\Y'%\$'k YY\_ hYfa 'd'i g'%\$!&\$'\ci fg'cZYI Ua 'dfYd"'H\Y'bcb!WcbhUWhh]a Y'YUW\'k YY\_'g\ci `X'VY'gdYbh]b'a U\_]b[ 'gi fY h\Uhmci 'i bXYfghUbX'h\Y''YWh fY ‰UhYf]U'žfYj ]Yk ]b[ 'fYgci fWYg'dfc|%XYX'cb'A ccX'Yž'df\$ % fY

		]



#### FYgci fW/g

#### DfYgWf]VYX F Ygci fWyg

H\YfY`]g`bc`hYI h`Vcc\_`Zcf`h\]g`Wci fgYž\ck Yj Yfž'hcd]WgdYW]Z]WfYgci fWYg`k]```VY`Uj U]`UV`Y`j ]U`A ccX`Y''`\\
UXX]h]cb`hc`[cj Yfba YbhfYgci fWYgž'Wcbgi `hUbhUbX`B; C`fYdcfhg`k]```cZhYb`VY`i gYZ\``Zcf`h\]g`Wci fgY''
9bYf[mDc`]Whi]g`h\Y`a cghfY`Yj UbhUWUXYa]Wci fbU`ž'UbX`fY`Yj UbhUff]WYg`U`gc`UddYUf`]b`ch\Yf`a cfY
hYW\bc`c[mZcW`gYX`ci fbU`g`gi W\`Ug`Gi ghU]bUV`Y`UbX`F YbYk UV`Y`9bYf[mFYj]Yk gžFYbYk UV`Y`9bYf[mž
5dd`]YX`9bYf[mžDfc[fYgg`]b`D\chcj c`hU]WgžK]bX`9bYf[mž`Gc`Uf`9bYf[mYW\

#### ?YYd]b[ i d hc XUhY

D7 ba {UkŽdWSfVW \_ WW [Sł dWbadfe S` V WhW fež

8 i f]b[ ``YWi fYg`UbX`h hcf]U`gža UhYf]U``k]```VY``]b\_YX`hc`W ffYbha YX]U`UbX`]ggi Yg`Ug`Uddfcdf]UhY''`Gh XYbhg

#### Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

 $K_cf_gi_Va_h^2X^uh'_k_h^2x^uh'_k_h^2X^uh'$ 

 $\label{thm:condition} $$HY^`UhY`dYbU`hmi]g`Udd`]YX`dYf`WU`YbXUf`XUmif]bWi X]b[`k YY_YbXg`UbX`di V`]W`\c`]XUmg&h\Uhih\Y UggYgga Ybhi]g`cj YfXi Y''`H\YfY`]g`bc`dfc!fUhU`cZh\Y``UhY`dYbU`hmiZcf`gi Va ]gg]cbg`a UXY`dUfhk Umih\fci [\'UXUm'$ 

K cf\_'gi Va ]hhYX'UZhYf'h\Y''XYUX`]bY'Zcf'UVgc`i hY'ZU]''']g'bchUWWdhYX'UbX'U'a Uf\_'cZnYfc'k]``'VY'Uk UfXYX Zcf'h\Uh'UggYgga Ybh']hYa "

cf'gca Y'UggYgga Ybh']hYa gž'U'``UhY'dYbU`hma Umbch'VY'Uddfcdf]UhY"'H\YgY'UfY'WYUf'm']bX]\WUhYX']b'h\Y Wci fgY'ci h']bYz'UbX'gi W\'UggYgga Ybhg'fYWY]j Y'U'a Uf\_'cZnYfc']Zbch'Wca d`YhYX'Vmh\Y'gdY\\JZ]YX'XUhY" 91 Ua d'Yg']bWi XY.

U"''K YY\_`mcb`]bY`hYghg`cf``UVcfUhcfmk cf\_`k cfh\`U`ga U```dfcdcfh]cb`cZh\Y`gi VYWha Uf\_žcf

V"''C b`]bY ei ]nnYg k \YfY Ubgk Yfg UfY fY`YUgYX hc gh XYbhg cb Wca d`Yhlcbžcf

····cf

X""DUgg# U]"UggYgga YbhHJg\_g"

#### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[`U`W`hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg`\UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{\WSeaXafZWbeS`VbSee[YfZW_aX\Sekagdai`ž$ 

D`U[]Uf]ga `]g`U`hmdY`cZ]bhY``YWi U``h\YZh'`±iWUb`HU\_Y`a UbmZcfa gžZfca `XY`]VYfUhY`W\YUh]b[`hc`UWV]XYbHU``m Wcdm]b[`Zfca `U`gci fWYk]h\ci hUW\_bck `YX[Ya Ybh'`I BGK `\Ug`dfcXi WYX`U`k YVg]hY`k]h\`U`k YU`h\`cZ fYgci fWg`hc`gi ddcfhighi XYbhg`hc`i bXYfgHUbX`UbX`Uj c]X`d`U[]Uf]ga žij ]g]h`ghi XYbh'i bgk "YXi "Ui #d`U[]Uf]ga "H\Y`@YUfb]b[`7 YbhfY`Ugg]ghg`ghi XYbhg`k]h\`i bXYfgHUbX]b[`UWUXYa]W]bhY[f]hmUbX`\ck 'bchhc'd`U[]Uf]gY" H\YmU`gc`\c`X`k cf\_g\cdg`UbX`WUb`\Y`d`ghi XYbhg`cbY!cb!cbY"

Mci 'UfY'U'gc'fYa ]bXYX'h\Uh'WUfYZ\ ``h]a Y'a UbU[Ya Ybh']g'Ub']a dcfhUbhdUfhcZgh XmUbX'cbY'cZh\Y ]XYbh]Z]YX'WUi gYg'cZd'U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi ZZ]W]Ybh'h]a Y'Zcf

#### 5 WUXYa ]W = 2cfa Uhlcb

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

: YYXVUW\_cb^h\Y`Wci fgY`]g`[ Uh\YfYX'dYf]cX]WU``mi g]b[ 'j Uf]ci g`a YUbgž]bWi X]b[ 'h\Y`I BGK a m9I dYf]YbW'`dfcWYggž]bZcfa U``X]gW gg]cb`]b`h\Y`Z]bU``WUgg`Zcf`h\Y`Wci fgYž'UbX`h\Y`GW\cc''g Gh XYbh#GHJZZa YYh]b[ g"'Mci f`ZYYXVUW\_`]g`H\\_Yb`gYf]ci g`mž'UbX`Wcbh]bi U``]a dfcj Ya Ybhg`UfY`a UXY`hc'h\Y Wci fqY`VUqYXž`]b`dUfhž'cb'qi W\`ZYYXVUW"

.

 $5 \text{ ``ghi XYbhg`UfY`Yl dYWhYX`hc`fYUX`UbX`VY`ZJa ]`]Uf`k ]h\`l BGK `[i]XY`]bYg`UbX`dc`]WYg''` ± b`dUfh]W\`Ufž ghi XYbhg`g\ci`X`VY`ZJa ]`]Uf`k ]h\`h\Y`Zc``ck ]b[.$ 

(E5 HY b X Ub W)

OH BGK '9a U] '5XXfYgg

UGdYWU`7cbg]XYfUhcb

(19) Ua q

(Enddfcj YX'7 U'W 'Uhcfg

(E5WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

(Bei ]hUV'Y'@YUfb]b['GYfj]WYa

=a U[Y'7fYX]h

Wcdmf][\h5bbU6fiW

7F = 7CG

7F = 7CG Dfcj ]XYf 7cXY. \$\$\$-,;

5W\_bck `YX[ Ya YbhcZ7ci bhfm

 $K_{Y'UW\underline{D}bck,YX[Y'h,Y'6YXY[U''dYcd'Y'k \c'UfY'h,Y'hfUX]h]cbU''W ghcX]Ubg'cZh,Y''UbXg'cb'k \] WX'I_BGK_{Ybg]b[hcb'WJa_di_g']g''cWJhYX'''$ 

# 5ddYbX]l. '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa +bhYbXYX @YUfb]b[ CihWca Yg	
?bck `YX[Y'UbX'g_]```VUgY	
D9%"%7cadfY\Ybg]j Yžh\YcfmVUgYX'i bXYfghUbX]b[ 'cZh\Y'i bXYfd]bb]b[ 'bUhi fU`'UbXd\mg]WU`'gW]YbWYg'UbX'h\Y'Yb[ ]bYYf]b[ 'Z bXUaYbhU`g'Udd`]WUV`Y'hc'h\Y'Yb[ ]bYYf]b[ 'X]gW]d`]bY	
D9%"&`7cbWfdhiU``ibXYfghUbX]b['cZh\Y'aUh\YaUh]WgžbiaYf]WU``UbU`mg]gžghUh]gh]WgžUbX WcadihYf`UbX`]bZcfaUh]cb'gW]YbWYg'k\]W\ibXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" '±0!XYdh\'i bXYfghUbX]b[ 'cZgdYW]U']ghVcX]Yg'cZ_bck 'YX[ Y'k ]h\]b'h\Y'Yb[ ]bYYf]b[ X]gW]d`]bY	
D9%"( '8]gWYfba YbhcZ_bck `YX[Y`XYjY`cda YbhUbX`fYgYUfW\XX]fYWIjcbg`k]h\]b`h\Y Yb[]bYYf]b[`X]gWJd`]bY	
D9%") "? bck `YX[Y`cZYb[]bYYf]b[ 'XYg][b'dfUWf]WY'UbX'WcbHYl hi U``ZUWrcfg']a dUWf]b[ 'h\Y Yb[]bYYf]b[ 'X]gW]d`]bY	
D9%"* `I_bXYfgHJbX]b[ `cZh\Y`gWcdYž'df]bW]d`Ygž'bcfa gž'UWVci bHJV]`]h]Yg`UbX`Vci bXg`cZ gi gHJ]bUV`Y`Yb[ ]bYYf]b[ `dfUWf]WY`]b`h\Y`gdYW]Z]WX]gW]d`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUh]cb`cZYgHUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j ]b[	