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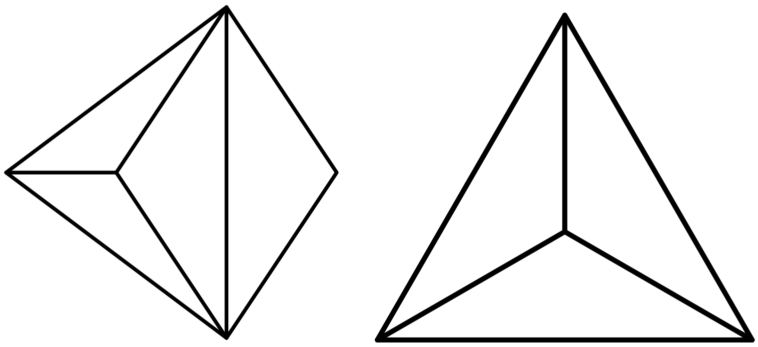
2016 UNSW School Mathematics Competition

Problem 4

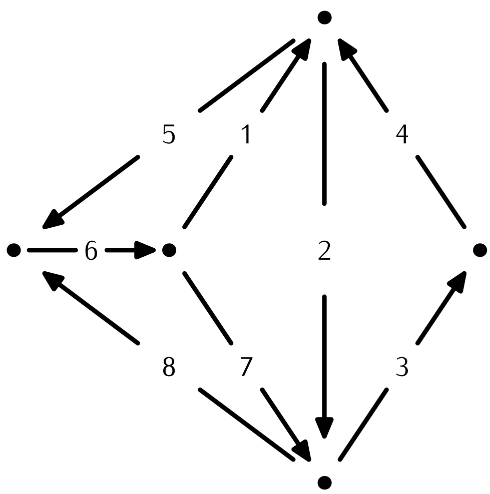
Is it possible to draw each of the following graphs in one move without lifting the pencil tip off the paper and without running through the same edge twice?



(a) (b)

Solution.

(a) It is possible. Here is one solution:



(b) It is not possible: if there was such a path, then it would have one start and one end vertex. These two would have an odd number of edges connected to them, whereas the rest of the vertices would be connected to an even number edges.

Solution.

2016 UNSW School Mathematics Competition
Senior Division – Problems and Solutions

Solutions by **Denis Potapov**[2]

Problem 1

A barrel has at least 10 L of gasoline. Gasoline can be removed and stored using a 5 L and a 9 L bucket. Explain how to remove exactly 6 L of gasoline from the barrel.

Solution. The following table present the steps of the solution. Here, $\mathbf{a} \geq 10$ is the initial amount of gasoline in the barrel:

#	Barrel	5 L bucket	9 L bucket
1	$\mathbf{a}$	0	0
2	$\mathbf{a} - 5$	5	0
3	$\mathbf{a} - 5$	0	5
4	$\mathbf{a} - 10$	5	5
5	$\mathbf{a} - 10$	1	9
6	$\mathbf{a} - 1$	1	0
7	$\mathbf{a} - 1$	0	1
8	$\mathbf{a} - 6$	5	1
9	$\mathbf{a} - 6$	0	6

Problem 2

You have a balance scale and four coins. You know that one coin is counterfeit but you do not know if the counterfeit coin is lighter or heavier.

(a) Explain how to identify counterfeit coin if you may only weigh coins twice.

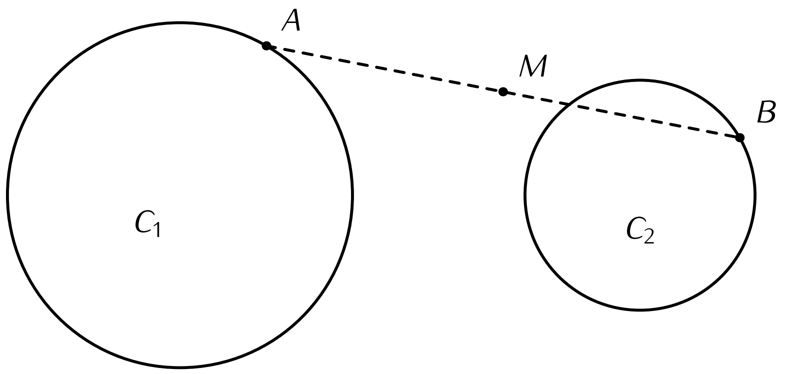
(b) Can you determine whether the counterfeit coin lighter or heavier?

Solution. Let us label the coins **A**, **B**, **C** and **D**.

(a) Assume first that we compared **A** and **B** and $\mathbf{A} \neq \mathbf{B}$ (e.g., $\mathbf{A} < \mathbf{B}$). In such a case, both coins **C** and **D** are true; the counterfeit coin is betwevere R21-20vern —446.28-1489989.89

Problem 5

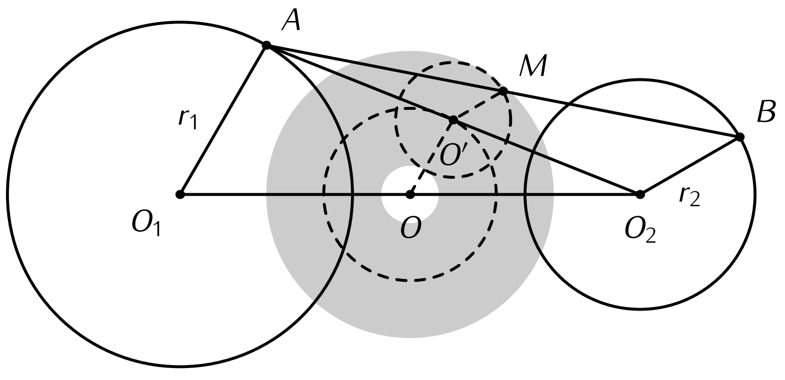
Let $\mathbf{A}$ be a point on the circle $\mathbf{C}_1$ and $\mathbf{B}$ be a point on the circle $\mathbf{C}_2$ as shown. Let $\mathbf{M}$ be the midpoint of $\mathbf{AB}$. As $\mathbf{A}$ and $\mathbf{B}$ move independently around each circle, describe in detail the curve traced by the point $\mathbf{M}$.



Solution. Let the centres and radii of the circles be $\mathbf{O}_1$ and $\mathbf{O}_2$ and $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. Assume that $\mathbf{r}_1 \geq \mathbf{r}_2$. Let $\mathbf{A}$ and $\mathbf{B}$ be random points on the circles and $\mathbf{M}$ be the middle point of $\mathbf{AB}$.

By connecting the points $\mathbf{A}$ and $\mathbf{O}_2$ and marking the midpoint of $\mathbf{AO}_2$ by $\mathbf{O}'$, we note that $\mathbf{OO}'$ is the mid-segment of the triangle $\triangle\mathbf{AO}_1\mathbf{O}_2$ and $\mathbf{O}'\mathbf{M}$ is the mid-segment of the triangle $\triangle\mathbf{AO}_2\mathbf{B}$. Hence, the set of points $\mathbf{M}$ can alternatively be obtained by running the point $\mathbf{M}$ over the the circumference of a circle with centre at $\mathbf{O}'$ and radius $\frac{r}{2}$ while the centre $\mathbf{O}'$ is run over the circumference of the circle with centre at $\mathbf{O}$ and radius $\frac{r}{2}$. The latter set is, in fact, an annulus with centre at $\mathbf{O}$ and inner and outer radii given by

$$\frac{\mathbf{r}_1}{2} - \frac{\mathbf{r}_2}{2} \quad \text{and} \quad \frac{\mathbf{r}_1}{2} + \frac{\mathbf{r}_2}{2}\,.$$



Problem 6