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56th UNSW School Mathematics Competition

Solutions by Denis Potapov[1]

A Junior Division – Problems

Problem A1:
In the country **Digit-land**, there are nine cities: “1”, “2”, . . . , “9”.
Two cities [illegible] and [illegible] are connected by a flight if and only if the two-digit number [illegible] is divisible by 3. Is there a way to travel by flights from the city “1” to the city “9”?

Problem A2:
A court hears the case of three suspects: Brown, Jones and Smith. One of them has committed a crime. Every suspect has made two statements, as follows.

Brown:	“I did not do it”, “Smith did it”;
Jones:	“Smith is innocent”, “Brown did it”;
Smith:	“I did not do it”, “Jones did not do it”.

The court has established that

(a) one of the suspects lied two times;

(b) one of the suspects told the truth two times;

(c) one of the suspects lied once and told the truth once.

Find out who has committed the crime.

Problem A3:
Show that the digits of a six-digit number can always be ordered so that the difference between the sum of the first three digits and the sum of the remaining three digits is less than 10.

Problem A4:
A plane is covered by an infinite square grid. Every square of the grid is painted by one of six colours. Prove that there are four squares of the grid painted with the same colour such that the centres of these squares form the corners of a rectangle with sides parallel to the lines of the grid. □

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Problem A5:
Two straight lines pass through two vertices of a triangle such that the triangle is cut into four smaller pieces: three triangles and a quadrilateral. It is possible to choose the lines such that the areas of these pieces are the same?

Problem A6:
There is a wolf at the centre of a square block of land. There is a dog located at each of the four vertices of the square. The wolf is allowed to move freely within the square and the dogs are only allowed to run by the sides of the square. Every dog is 50%

Problem B4:
Every point of a plane is painted with one of two different colours such that every equilateral triangle of side 1 has two of its vertices painted with different colours. Prove that there is an equilateral triangle with side $\sqrt{3}$ such that its every vertex is painted with the same colour.

Problem B5:
There is a rabbit in the centre of a square block of land. There is a wolf located at each of the four vertices of the square. The rabbit is allowed to run freely within the square and the wolves are only allowed to run by the sides. Every wolf is 40% faster than the rabbit. Is there a strategy for the rabbit to escape the square?

Problem B6:
Let [illegible] be a parallelogram and let the bi-sector of the angle $\angle$ [illegible]

Solution A4.

Choose any horizontal seven-squares-high strip on the plane. Note that there are finitely many different ways to paint squares of a one-square-wide and seven-squares-high column. Hence, there are two such columns in the strip we chose earlier which are painted with identical sets of colours. Since, we only have six colours and the columns have seven squares, there will be two squares in each column painted with identical colours. These squares make the rectangle as required. □

Solution A6.

Run two lines through the position of the wolf parallel to each of the diagonals of the

□

Solution B3.

Let the first friend in the conversation be F1 and the second friend be F2.

If F1 is “Bob”, then he tells the truth in his first reply. In his

Solution B5.

Solution B6.

The latter is sufficient to claim that To prove that the quadrilateral is circumscribed, we just need to prove that ∠ = ∠ .

Since ◢ is a bi-sector, it follows that ∠ ◢ = ∠ ◢. Furthermore, we also have ∠ = ∠ = ∠ ◢. Consequently, = and = ◢. Now, △ = △ since the corresponding sides are equal. Hence, ∠ = ∠ .

If we rotate the segment around point by the angle ∠ , then the point rotates to the point ; the point rotates to the point . Hence, ∠ is the angle between and , namely the angle of rotation.

On the other hand, since = ◢ = , the point rotates to the point . Hence, ∠ is the angle of rotation. That is, ∠ = ∠ .