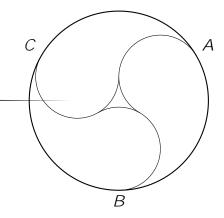
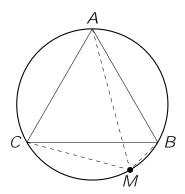
MATHEMATICS ENRICHMENT CLUB. Problem Sheet 18, September 24, 2018

- 1. A chess board is an 8 8 grid of squares coloured white or black so that no two adjacent squares are the same colour. Given tiles that are 2 1 grid squares, it is possible to cover the chessboard completely, and it takes precisely 32 tiles. Show that it is impossible to cover a chessboard with opposite corners removed.
- 2. Find all 3 digit numbers which are equal to the sum of the factorials of their digits.
- 3. In the diagram below, ABC is a circle of radius R with 3 tear-drop shapes inside. Each of the arcs $AC^{\emptyset}A^{\emptyset}$, $BA^{\emptyset}B^{\emptyset}$ and $CB^{\emptyset}C^{\emptyset}$ are of circles of the same radius, r. Find the area of each tear drop in terms of r.



5. The point M lies on the circumcircle of the equilateral triangle 4ABC, as shown in the diagram.



Prove that AM = MB + MC.

Senior Questions

1. Let f and g be real-valued, continuous functions de ned on 1 x 1. We de ne the *inner product* of f and g, hf; gi, as

$$hf;gi = \int_{1}^{Z} f(x)g(x) dx:$$

Consider the polynomials $p_0(x) = 1$ and $p_1(x) = x$.

- (a) Two functions, f and g, are said to be *orthogonal* if hf; gi = 0. Show that p_0 and p_1 are orthogonal.
- (b) A function f is said to be *normalized* if hf; fi = 1. Find factors $_0$ and $_1$ such that the polynomials $q_0 = _0p_0$ and $q_1 = _1p_1$ are not only orthogonal but also normalized.
- (c) A set of polynomials that are all normalized and mutually orthogonal is called