MATHEMATICS ENRICHMENT CLUB. Problem Sheet 13, August 27, 2019

- 1. Suppose that x is a three-digit positive integer. The six-digit number y is created by repeating the digits of x. If y is also a multiple of x^2 , nd the values of x and y.
- 2. Find the last digit of $1^5 + 2^5 + \dots + 2019^5$.
- 3. Two congruent semi-circles of radius *R* are drawn inside a larger semicircle. A smaller circle, with radius *r*, is also inscribed in the larger semi-circle so that it is tangent to all three semicircles, as shown in the diagram below.



Show that R: r = 3:2.

- 4. Integers 1;2;:::;100 are written in a circle, not necessarily in that order. Can it be that the absolute value of the di erence between any two adjacent integers is at least 30 and at most 50?
- 5. Consider an arbitrary number a > 0. We know that the inequality $10 < a^x < 100$ has exactly 5 positive integer solutions for x. How many solutions in positive integers may the inequality $100 < a^x < 1000$ have? In each case, list the solutions.

Senior Questions

1.