MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 10 Solutions, August 13, 2019

1. Carla wins if either or both players rolls a 5 or 6. Summing the probability of each case to occur gives

$$2 \quad \frac{2}{6} \quad \frac{4}{6} \quad + \frac{2}{6} \quad \frac{2}{6} = \frac{5}{9};$$

   Carla is more likely to win.

2. First note that $2^{10} = 1024$, so that the distinct numbers we are adding can have at most a power of 9 on 2. Now adding the ten possible distinct powers of 2, gives $2^0 + 2^1 + \ldots + 2^9 = 2^{10} \quad 1 = 1023$, and we have to delete a number from the summation

Now $x \neq y$, so we can cancel the common factor of $\sqrt{x} - \sqrt{y}$ to obtain

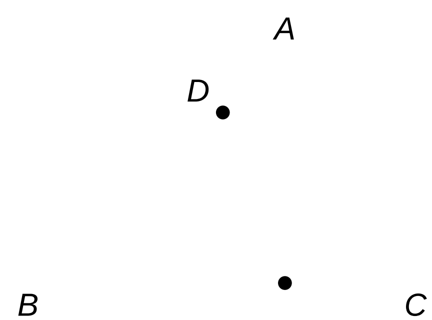
$$\sqrt{x} + \sqrt{y} = 1.$$

Squaring both sides and rearranging, we have

$$\begin{aligned} x + 2\sqrt{xy} + y &= 1 \\ x + y &= 1 - 2\sqrt{xy} \end{aligned}$$

Now the LHS is what we are trying to maximise. If we look at the RHS, we can see that this is maximised if either x or y is zero and the other number must then be one. So the maximum value is one.

5. Let $\angle BAC =$ and $\angle ABC =$.



Senior Questions

1. Since we are dividing by $x^2 \quad 1$, the remainder is a polynomial of x of at most degree 1; that is the remainder takes the form $ax + b$, for some constants a and b.

   To nd a and b, write

$$
\begin{aligned}
x^{2019} &= Q(x)(x^2 \quad 1) + ax + b \\
&= Q(x)(x \quad 1)(x + 1) + ax + b;
\end{aligned}
$$

   where $Q(x)$ is a polynomial of x. Then by putting $x = 1$ and $x = \quad 1$ into the last line of the above equation, we have $a + b = 1^{2019} = 1$ and $\quad a + b = \quad 1^{2019} = \quad 1$. Solving these simultaneously, we arrive at $a = 1$ and $b = 0$.

2. By the sine rule,

$$
\begin{aligned}
\frac{\sin \quad}{4} &= \frac{\sin 2 \quad}{6} \\
3 \sin \quad &= 2 \sin 2 \\
3 \sin \quad &= 4 \sin \quad \cos \quad ;
\end{aligned}
$$

   so

$$
\sin \quad = \frac{4}{3}
$$