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MATHEMATICS ENRICHMENT CLUB.

Problem Sheet 14 Solutions, September 10, 2019

1. Suppose we take four socks and none of them match, then the next sock we draw must

16 multiples of 125; and

3 multiples of 625

So the answer is 403 + 80 + 16 + 3 = 502.

5. Observe that $y = 0; x = 1; p = 2$ and $y = 1; x = 1; p = 5$ are two possible solutions. We show that there is no other. Assume $y > 0$, and write $p^x = y^4 + 4 = y^4 + 4y^2 \quad 4y^2 + 4 = (y^2 \quad 2y + 2)(y^2 + 2y + 2)$. Then $y^2 \quad 2y + 2 < y^2 + 2y + 2$, which implies $y^2 + 2y + 2$ is divisible by $y^2 \quad 2y + 2$ because their product is a power of the prime number p. On y22

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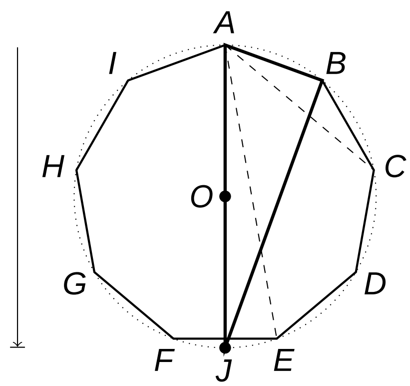
Senior Questions

1. Without loss of generality, we can inscribe the regular nonagon in a circle centered at O with diameter 1 unit. Let J be the point on the circle opposite A, so that AOJ is a diameter of the circle. Then AJ is one unit.



i. $b + c \quad d$. Then each of six triples in which two numbers are from the set $fa; b; cg$ and the third number is from the set $fd; e; g$ does not form a triangle.

ii. $c + d \quad e$. Then each of six triples which includes e does not form a triangle.

iii. $b + d \quad e$ and $a + b \quad d$. Then each of six triples $fa; b; dg$, $fa; b; eg$, $fa; c; eg; fa; d; eg\ fb; c; eg$, $fb; d; eg$ does not form a triangle.

Suppose that neither of above cases takes place, that is, $b + c > d$, $c + d > e$ and at least one of inequalities $b + d > e$ and $a + b > d$ holds. We shall show that this is impossible.

iv. If $b + c > d$, $b + d > e$ then

$$a2 + b2 + c2 + d2 + e2 < ab + bc + ce + (b + c)d + (b + d)e:$$

Contradiction.

v. If $c + d > e$, $a + b > d$ then

$$a^2 + b^2 + c^2 + d^2 + e^2 < ab + bc + cd + (a + d)d + (c + d)e:$$

Contradiction.