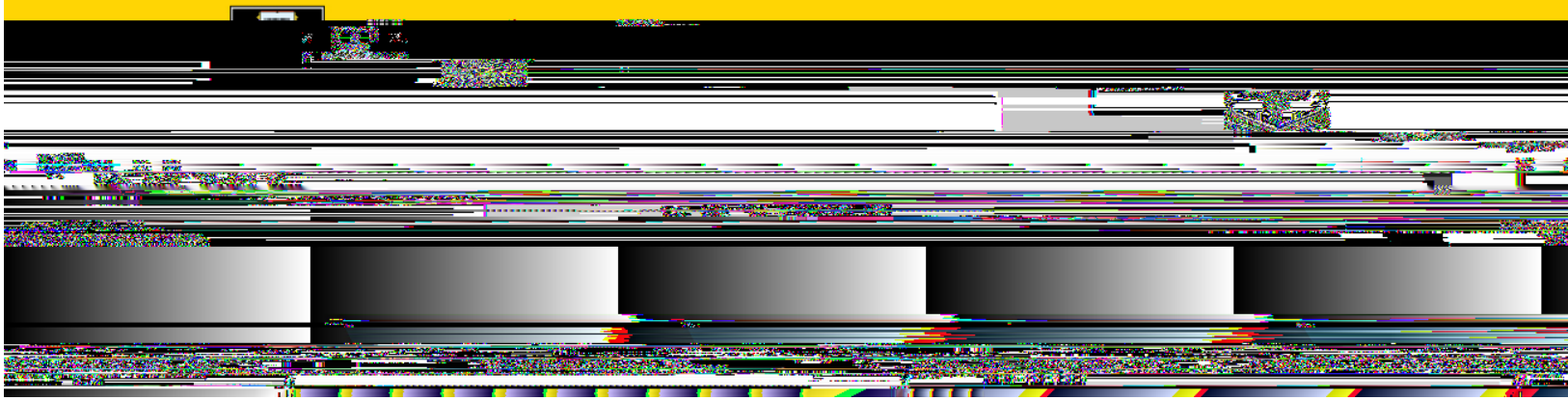

MATHEMATICS ENRICHMENT CLUB.[1]

Problem Sheet 10, July 30, 2012

1. Starting from zero, what is the 2012th palindromic number?

2. What is the average of 35 successive positive odd numbers beginning with 7?

3. Can you make up some examples in which $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$?

4. (a) Show that both 29 and 37 can be written as the sum of two squares, but that 30 and 31 cannot.

   (b) Show that $(a^2 + b^2)(c^2 + d^2) = (ac \quad bd)^2 + (ad + bc)^2$

   (c) Use the formula in (ii) to show how to write $1073 = 29 \quad 37$ as the sum of two squares. In how many ways can 1073 be written as the sum of two squares?

5. 10 darts are thrown onto a square dart board which is 3m by 3m. Prove that at least two of the darts land within $p\overline{2}$ m of each other.

6. Given two intersecting lines ` and $m$ and a point $P$ not on either line, show how to construct a straight line which passes through $P$ meeting ` and m in points B and C respectively such that:

   (a) $BP = PC$

   (b) $BP : PC = 1 : 3$:

7. Two circles $C_1; C_2$ with centres O_1 and O_2 are externally tangent at P. Let $A; B$ be points on each circle such that AB is a common tangent to both $C_1; C_2$.

   Suppose4.733 1.794 Td [(;)-1g89913common tangent to1eet-326(a)t9Bt1-1gG.an(a)t9B326(tangen)27

(c) Given the radii of the two circles are respectively 8cm and 2cm, nd the length O_1X.

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.