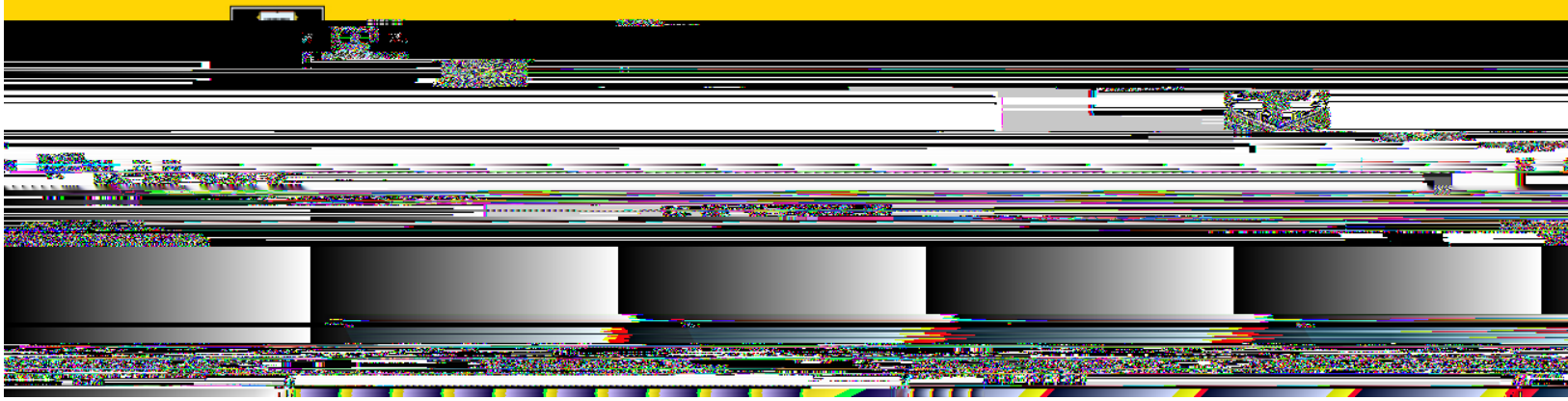

MATHEMATICS ENRICHMENT CLUB.[1]

Problem Sheet 15, September 3, 2012

1. In how many ways can we change \$10 into 50 cent and 20 cent coins, with at least one of each coin being used.

2. If $x = \sqrt{1 + \sqrt{1 + \sqrt{2}}}$ nd the exact value of $x^4 \quad 2x^2$.

3. A quadrilateral in which a circle can be drawn which touches each of the four faces is called a *circumscribable quadrilateral*. If r is the radius of the circle and s is half the perimeter of the quadrilateral, prove that the area of the quadrilateral is rs.

4. Use the fact that $2xy = (x + y)^2 \quad x^2 \quad y^2$ to show that

$$2(b \quad c)(c \quad a) + 2(c \quad a)(a \quad b) + 2(a \quad b)(b \quad c) \quad 0$$

for all real numbers $a; b; c$.

5. (a) Find all positive integers $a; b; c$ such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is as large as possible but less than $\frac{1}{2}$:

(b)

2. Prove by induction that the sum to k terms of

$$1^2 \quad 3^2 + 5^2 \quad 7^2 + \ldots\ldots$$

equals $8n^2$ when $k = 2n$ and $8n^2 + 8n + 1$ when $k = 2n + 1$.

3. In ABC prove that $b^2(\cot A + \cot B) = c^2(\cot A + \cot C)$. (Hint: You might begin by considering the area of the triangle in two di erent ways.)