MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 3, May 14, 2012

1. The perimeter of a base of a rectangular brick with integer sides is 18 cm, whilst its volume is 42 cm^3. What is its height?

2. Calculate

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 - \frac{1}{2008}\right):$$

3. Find the smallest positive integer whose square ends in (a) 09 and (b) 9009.

4. Show that if a, b are positive numbers such that $ab \leq 1$ then

$$\frac{a}{b+1} + \frac{b}{a+1} + (1-a)(1-b) \leq 1:$$

5. Suppose we have the numbers $x_0 = 0, x_1 = 1$ and $x_{n+1} = x_n + 2x_{n-1}$ for $n \geq 2$.

a. Write down the numbers x_n for $n = 2, 3, 4, 5, 6$.

b. Show that there is no n for which x_n

Senior Questions.

1. Find $\displaystyle\int_0^1 \frac{1}{1 + t + t^2}$.

2. Find the limit $\displaystyle\lim_{n!} \frac{1}{1}$