.[1]

, 21, 2012

1. If $f(n) = (n \quad 1)f(n \quad 1)$ and $f(1) = 1$ nd $f(4)$.

2. The product of the ages in years of two adults is 770. What is the sum of their ages?

3. (a) How many positive integers are there 100 which have no factors, except 1, in common with 100?

   (b) What is their sum?

4. If $x_1 = 3$, the recurrence $x_{n+1} = x_n^2 \quad 10x_n$, gives the sequence $3; \quad 21; 651; 417291:::$ and the numbers increase without bound. Find all the values for x_1 so that the sequence does NOT increase without bound.

5. Solve the simultaneous equations:

$$x + yz = 2$$

$$y + xz = 2$$

$$z + xy = 2:$$

6. Two circles $C_1; C_2$ with centres $O_1; O_2$ are externally tangent at the point P. A straight line through P meets $C_1; C_2$ respectively at A and B. Show that the tangents to the circles at A and B are parallel.

7. Let $ABCD$ be a trapezium with $AB jj CD$. Let P be the intersection of the diagonals AC and BD.

   (a) Show that the triangles APD and BPC have the same area.

   (b) Given that APB has area 1 cm^2 and that APD has area 4 cm^2, nd the area of $ABCD$.

[1] Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

1. Find $\int \frac{1}{x + \sqrt{x}}\, dx.$

2. Find $\lim_{n\to\infty} \frac{n!}{n^n}$.

3. Prove that

$$1 \cdot 3 \cdot 5 \cdots (2n - 1) = \frac{(2n)!}{2^n n!}.$$