

MATHEMATICS ENRICHMENT CLUB.¹ Problem Sheet 5, May 28, 2012

- 1. Two classes of 20 and 30 students average 66% and 56% respectively on an examination. What is the average for all the students on the exam?
- 4. Find in nitely many integers x such that

$$\sqrt[3]{}$$

$$\frac{1}{X + P_{\overline{X^2 + 1}}} + \sqrt[3]{X - P_{\overline{X^2 + 1}}}$$

is an integer.

- 5. (a) Prove that $a + b = 2^{p} \frac{1}{ab}$ for any positive real numbers a; b.
 - (b) Deduce that for x; y; z positive, (x + y)(x + z)(y + z) 8xyz:
- 6. In the triangle ABC, it is given that $\angle ABC = 140$. Let D be a point on AC and E a point on AB such that the three triangles AED; EDB and DBC are all isosceles, with their vertices at E; D and B respectively. Find all the angles of the triangle ABC.
- 7. Let ABCD be a trapezium and with ABjjCD. Let M;N be the midpoints of AD and BC respectively. Show that $MN = \frac{1}{2}(AB + CD)$:

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

Senior Questions.

1. Let
$$f(x) = \left(1 + \frac{1}{x}\right)^x$$
.

- (a) Prove that $\frac{f^{\emptyset}(x)}{f(x)} = \log\left(1 + \frac{1}{x}\right) = \frac{1}{1+x}$:
- (b) By considering the area under the curve $y = \frac{1}{t}$ for t from 1 to 1 + $\frac{1}{x}$, show that $\log\left(1 + \frac{1}{x}\right) > \frac{1}{1+x}$ and deduce that f(x) is increasing.
- 2. Suppose a > b > 0. Find $\lim_{n! \to 1} (a^n + b^n)^{\frac{1}{n}}$.
- 3. By considering cos(A + B) + sin(A B) = 0 nd the general solution (for) of cos n + sin m = 0.