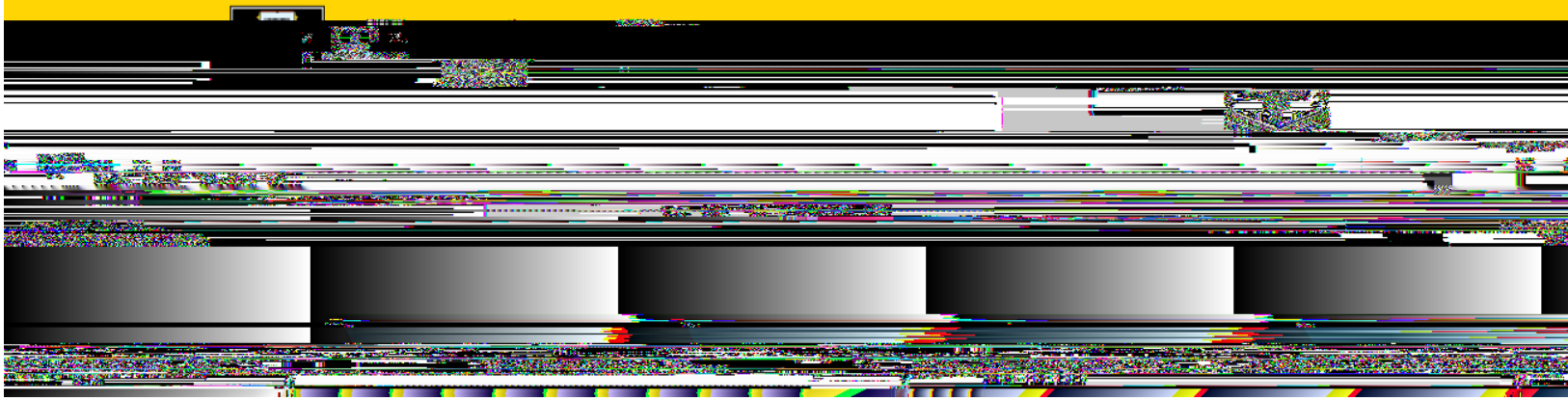

MATHEMATICS ENRICHMENT CLUB.[1]

Problem Sheet 7, June 9, 2012

1. In how many ways can 6 boys and 6 girls stand next to each other in a row such that no two boys stand next to each other and no two girls stand next to each other?

2. The notation 5! means $5 \times 4 \times 3 \times 2 \times 1 (= 120)$. How many zeros are there at the end of 1000!.

3. (a) $a; b$ are positive numbers with $a + b = k$. Explain why ab is greatest when $a = b = \frac{k}{2}$:

   (b) Suppose that $x^2 + y^2 = c^2$, nd the minimum value of $x^4 + y^4$.

4. (a) Show that there are in nitely many non-zero integers $x; y; z$ such that $2^x + 2^y = 2^z$:

   (b) Show that if $n > 2$ then there are no nonzero integers $x; y; z$ such that $n^x + n^y = n^z$:

5. (Parts b and c require Year 9 and Year 10 Mathematics).
   Let ABC be an isosceles triangle with the base angles B and C being 72° and $AB = AC = 4$. The length of the base BC, called x is chosen such that a line CD can be drawn, where D lies on AB, such that $\angle BDC = 72^\circ$.

   (a) Find a pair of similar triangles and show that x satis es, $x^2 + 4x - 16 = 0$:

   (b) Use triangle ABC to nd $\cos 72^\circ$ in surd form.

   (c) Use triangle ACD to nd $\cos 36^\circ$ in surd form.

6. Suppose that two non-parallel straight lines k and ℓ meet at a point P which is not on the page of my book. Construct a line which would (if P did lie on the page) bisect the angle between the lines and pass through P.

7. Let $K; L$ be points on the sides $AB; AD$ respectively of the convex quadrilateral $ABCD$ such that $AK = \frac{1}{3}AB$ and $AL = \frac{1}{3}AD$: Similarly, $M; N$ are points on $CD; CB$ such that $CM = \frac{1}{3}CD$ and CN

Year 11 Question.

1. Suppose that m and n are positive real numbers. Use trigonometry to find the the maximum value of
$$\frac{m+n}{\sqrt{m^2+n^2}}.$$