MATHEMATICS ENRICHMENT CLUB.[1]
Solution Sheet 10, July 30, 2013

1.

$$(x^{-1} + y^{-1})^{-1} = \frac{1}{\frac{1}{x} + \frac{1}{y}} = \frac{1}{\frac{y+x}{xy}} = \frac{xy}{y+x}.$$

2. For a number to be a cube it's prime factorisation must contain only cubes. The prime factorisation of $60 = 2^2 \times 3 \times 5$ so $n = 2 \times 3^2 \times 5^2 = 450$.

3. Using long division we can see that 12 950 264 876 is divisible by 3 but that $4\,650\,088\,292 = \frac{12\,950\,264\,876}{3}$ is not. Thus the prime factorisation of 12 950 264 876 contains a 3 which is not squared, so cannot be a square.

4. There is a multiplier α such that the angles of our triangle are 2α, 3α and 4α. Using the angle sum $2\alpha + 3\alpha + 4\alpha = 180 \implies \alpha = 20$. So the angles are 40, 60 and 80.

5. Let the median from C to AB meet AB at D. Since DC has length $\frac{1}{2}AB$

Now we divide $2^{20} + 1$ by 2^{10}

2. Using various angle expansions obtain

$$
\begin{aligned}
\cos((n + 2)\ \ ) &= \cos((n + 1)\ \ )\cos \qquad \sin((n + 1)\ \ )\sin \\
&= \cos((n + 1)\ \ )\cos \qquad \sin\ \ (\sin(n\ \ )\cos\ \ + \cos(n\ \ )\sin\ \ ) \\
&= \cos((n + 1)\ \ )\cos \qquad \sin(n\ \ )\sin\ \ \cos \qquad \cos(n\ \ )\sin^2(\ \ ) \\
&= \cos((n + 1)\ \ )\cos \qquad \frac{1}{2}\sin(n\ \ )\sin(2\ \ ) \qquad \cos(n\ \ )\sin^2\ \ :
\end{aligned}
$$

Now note

$$
\begin{aligned}
\sin(n\ \ )\sin(2\ \ ) &= \cos(n\ \ )\cos(2\ ) \qquad \cos(n\ \ + 2\ \ ) \\
&= \cos(n\ \ )(2\cos^2 \qquad 1) \qquad \cos((n + 2)\ \ ):
\end{aligned}
$$

So

$$\cos((n + 2)\ \ ) = \cos((($$