MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 12, August 12, 2014[1]

1. (a) At the end of 30! there are 7 zeros. What is the digit to the left of these 7 zeros?

   (b) How many zeros does 1000! have at the end?

2. (a) Start with an equilateral triangle coloured red on a white background. Divide it into 4 congruent equilateral triangles and erase the red inside the centre triangle. How many equilateral triangles are there using only lines which separate red from white?

   (b) Do the same for each of the remaining red triangles. Again, how many equilateral triangles are there using only lines which separate the red from white?

   (c) Suppose you've done this process n times. How many equilateral triangles are there using only lines which separate red from white?

   (d) Suppose you've done this process n times. What is the ratio of the new red area to the original red area?

3. The positive integer d is not 2, 5 or 13. How many values of d are there such that, for any pair, (a, b), of numbers from $\{2, 5, 13, d\}$, $ab - 1$ is a perfect square?

4. How many ordered pairs, (a, b), are there of positive integers a and b between 1 and 999 (inclusive) such that

$$(a + 36b)(b + 36a)$$

   is an integral power of 2?

5. Let $A_1, A_2, \ldots, A_{2n}$ $(n \geq 2)$ be $2n$ ordered points equally spaced around a circle such that $A_1A_2\ldots A_{2n}$ is a regular polygon.

   Any 3 points form a triangle while only some sets of 4 points form rectangles. Given that there are 20 times more triangles than rectangles find n.

6. The segments that connect the midpoints of opposite sides of a convex quadrilateral $ABCD$ divide it into 4 quadrilaterals of the same perimeter. Prove that $ABCD$ is a parallelogram.

[1] Some problems from UNSW's publication Parabola, others from www.brilliant.org