MATHEMATICS ENRICHMENT CLUB. Problem Sheet 8, June 24, 2014¹

- 1. Website *A* demands its users have a password that is 6{8 characters long, contain only letters or numbers and must contain at least three letters and three numbers but is case insensitive, while website *B* demands only that its users have passwords that are no more than 6 characters long, use only letters or numbers and is case sensitive. Which website has more possible passwords?
- 2. A thousand coins lie in one long row, all facing tails up. We rst walk along the line and ip over every coin. Then we start again, ipping over every second coin, then again ipping over every third coin, then again ipping every forth and so on. After doing this a thousand times, so that the last time we do it, we ip only the last (one thousandth) coin, how many coins are facing heads up?

the smallest number. Show that the arithr

3. Three primes play by the equation

$$p^2q^2 + r^2 = 2pqr + pq + r$$
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¹Some problems from UNSW's publicationarabola

 $\underline{z}_{n \text{ times}}$ as a n and

 $a = a_{\{z = a\}}$ as a^n . Let's keep going and write a^{a} .

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Senior Questions

- 1. A right cone is a cone whose apex lies exactly above the centre of the base. The volume of a right cone is $\frac{1}{3}bh$ where *b* is the area of the base, and *h* the perpendicular height of the cone. An oblique cone is a cone whose apex does not lie exactly above the centre of the base. Show that the volume of an oblique cone has the same formula as a right cone.
- 2. A glass in the shape of an inverted, truncated, right cone (i.e. frustrum) contains some water. The top of the glass has radius R, the bottom has radius r, r < R, and the glass is h tall. When tilted, the surface of the water can be made to reach from the tip