

MATHEMATICS ENRICHMENT CLUB.  
Problem Sheet 9, July 22, 2014 <sup>1</sup>

3. Archimedes weighs 60 kilograms. The earth weighs  $610^{24}$  kilograms. Archimedes is given a fulcrum and lever. The Earth is 2 meters away from the fulcrum. How far away does Archimedes have to stand, so that his weight moves the earth?

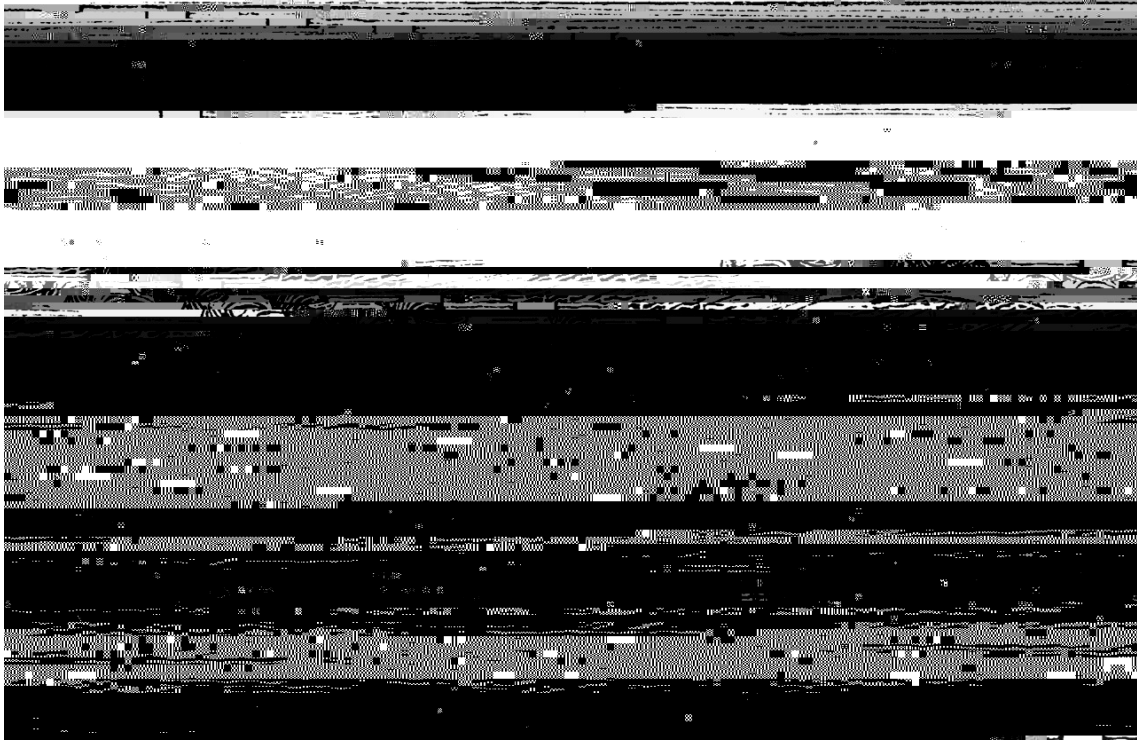


Figure 2: Give me but a rm spot on which to stand, and I shall move the earth - Archimedes

4. The levers in the figure below balance. Given  $d_1 : d_2$  and  $d_3 : d_4$  are in the ratio 1 : 2:
- What is the total mass supported by the smaller lever?
  - Given  $m_1 = 6$  kilograms, calculate the masses  $m_3; m_4$ .
  - Construct a new law of the lever with one fulcrum and three masses.

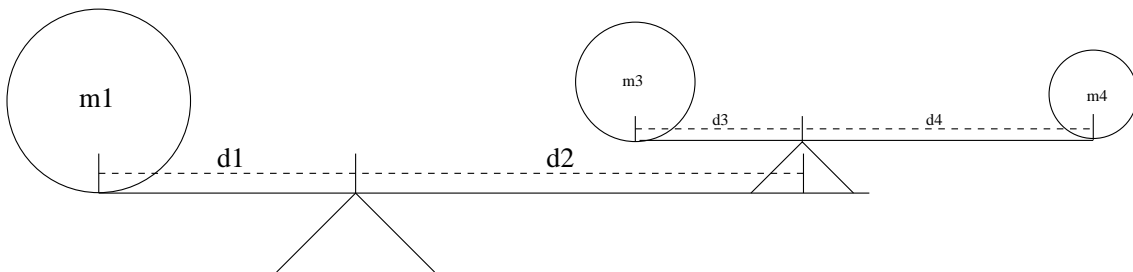
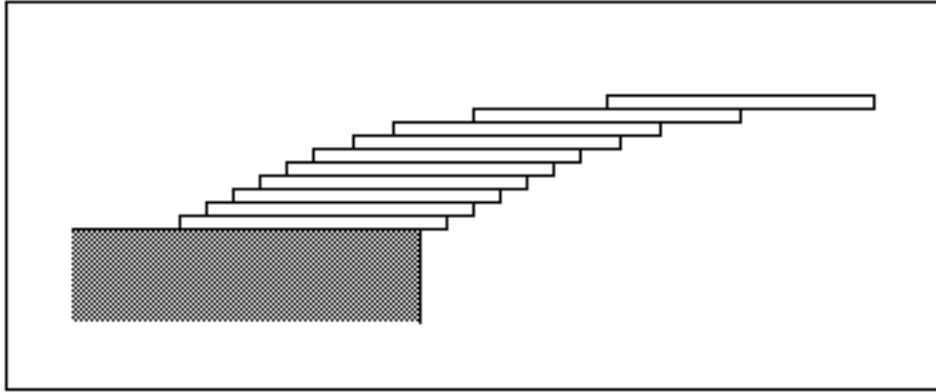


Figure 3: levers on levers

5. Given an infinite number of bricks of length 2 and equal mass.



- (a) How many bricks do you need to stack until the top one completely overhangs the bottom?
  - (b) Verify your answer using the objects around you.
  - (c) (challenge) how far can the bricks overhang?
6. Consider a (weightless) triangle ABC.
- (a) Place weights of mass 1 at A and B. Where is the centre of mass? (i.e. where is the fulcrum that balances this triangle?)
  - (b)

9. The centres of mass of the Earth ( $E$ ) and Moon ( $M$ ) orbit each other about an invisible fulcrum, called the barycentre  $B_{EM}$ .
- Given that the distance  $EM$  is 384 000km and the distance  $EB_{EM}$  is 4,670km, calculate the mass of the Moon relative to the mass of the Earth.
  - Let  $S$  be the centre of mass of the Sun. Given that the sun is 3330 times heavier than  $E$ , where is the Earth-Sun Barycentre  $B_{ES}$ ?
  - The centres of mass of Pluto  $P$  and Charon  $C$  are 19 600km apart, and the distance from  $P$  to its barycentre  $B_{PC}$  is 2,110km. Calculate the mass of Pluto relative to Charon.

#### Senior Question

In 2012, a powerful spectrograph measured a small change in the blue-shift of Alpha Centauri B ( $B$ ) with maximum velocity 0.5 meters per second (you could walk faster). This wobble had a period of  $28 \times 10^4$  seconds. As in question 9, wobble indicates the presence of an unseen companion called Alpha Centauri Bb ( $Bb$ ). The centres of mass of  $B$  and  $Bb$  are  $6 \times 10^6$ km apart.

- Draw a diagram showing the orbit of  $Bb$  about Alpha Centauri B, and Earth.
- Show that the barycentre  $B_{BBb}$  is at least 22km away from  $B$  (approx to the nearest km).
- Given that the mass of  $B$  is  $3 \times 10^5$  "Earth Units". Show that minimum mass of  $Bb$  is 1:1 Earth Units (about one Earth).