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MATHEMATICS ENRICHMENT CLUB.

Solution Sheet 4, May 27, 2014 [1]

1. Suppose we have a pizza of radius r that ts perfectly into a square box. The box's sides must then be $2r$, so the ratio of pizza area to box area is 372f6r

where $C(n; m)$ is the number of ways of choosing m things from n, so here we choose 1 of the thirteen numbers, then choose all 4 of those 4 cards and for our final card choose 1 from the remaining 48. The total number of hands is the number of ways of choosing 5 cards from 52.

We can count $P(B|A)$ a number of different ways, but I'll use the formula

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

where $P(B \cap A)$ means the probability of both B and A occuring. To get both players to have a 4 of a kind, we compute

$$P(B \cap A) = \frac{C(13; 1)C(4; 4)C(48; 1)C(11; 1)C(4; 4)C(43; 1)}{C(52; 5)C(47; 5)}.$$

That is, from the 13 numbers choose 1, then choose 4 of those 4 cards, and 1 from the remaining 48 for the first player's hand. Then for the second player, choose 1 of the 11 remaining numbers (remember the fifth card in player 1's hand can't be used to form a 4 of a kind), choose 4 of those 4 cards and then 1 from the remaining 43. The total number of hands is to first choose 5 from 52 for player 1 and then 5 from the remaining 47 for player 2.

The probability, $P(A)$ is the same as $P(B)$. So

$$P(B|A) = \quad C(13;$$

Senior Questions Consider the function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{for } x \neq 0; \\ 0 & \text{for } x = 0 \end{cases}.$$

1. Here all we do is check

$$\lim_{x \to 0} f(x) = f(0):$$

   The value of f at $x = 0$ is given by the bottom branch so $f(0) = 0$. For all $x \neq 0$, $-1 \leq \sin \frac{1}{x} \leq 1$, so $\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$.

2. Here we must check that

$$\lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

   exists. That is

$$\lim_{h \to 0} \frac{h^2 \sin \frac{1}{h}}{h}$$

   exists. Again, since $\sin \frac{1}{h}$ is bounded between -1 and 1 for all $h \neq$