MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 11, July 28, 2015[1]

1. Alice and Carla are playing a dice game. Here's how it works:

   Each person rolls a die, and the highest number rolled of the two is recorded.

   If the highest number rolled is a 1; 2; 3 or 4, Alice wins.

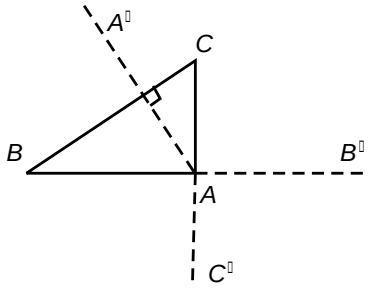
   If the highest number rolled is a 5 or a 6, Carla wins.

   On average, who is more likely to win: Alice, Carla, or are the probabilities equal?

2. Find the remainder when x^{1999} is divided by x^2 1.

3. How many 3 digit positive integer is/are the sum of exactly 9 distinct powers of 2?

4. Given that $a + b = 1$ and $a^2 + b^2 = 2$, what is the value of $a^7 + b^7$?



5. Let $\triangle ABC$ be right-angled. Let A' be the mirror image of the point A in the side BC, let B' be the mirror image of B in AC and C' the mirror image of B in AB; see above. Find the ratio

$$\text{area}(\triangle ABC):\text{area}(\triangle A'B'C').$$

6. Find all positive integers n for which all of the numbers

$$n;\ 2n - 1;\ 2n + 5;\ 3n - 2;\ 5n - 4;\ 6n - 5;\text{ and } 12n + 5$$

   are prime. (Note the integer 1 is not prime).

[1]Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*.

Senior Questions

1. Find all solutions of $2^x + 3^x + 6^x = x^2$.
2. Let $f(x) = x + \int_0^1 (xy^2 + x^2y) f(y)\, dy$. Find the value of $f(10)$.
3. Denote by $[a;[]$ *Find*