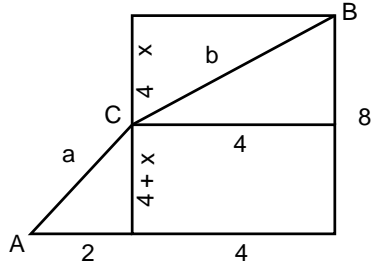


or. Let  $a$  be the greatest number written on a blackboard, pick another integer  $b$  on the board, then  $a > b$ . Furthermore, there is an integer  $n \geq 0$  such that  $2^n \leq a < 2^{n+1}$ , so that  $2^n < a + b \leq 2a < 2^{n+2}$ . Since  $a + b$  must be a power of two, we must have  $a + b = 2^{n+1}$ .

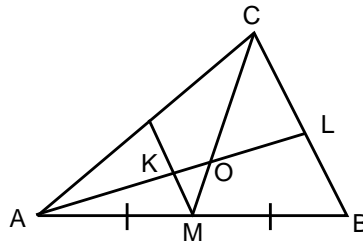
$2^2 = 16 + 4 = 32 + \dots$   $n$ -digit  
 contributes to the sum by an amount of  $(n-1)! (1 + 2 + 3 + \dots + n)$ .

Make a similar argument by fixing the second last digit, so that the second last digit of all combinations of  $n$ -digits long numbers contributes to the sum by an amount of  $(n-1)! (1 + 2 + 3 + \dots + n) \cdot 10$ .

Doing this for all digits, then the total sum is  $(n-1)! (1 + 2 + 3 + \dots + n) (10^0 + 10^1 + \dots + 10^{n-1})$ .



4. Consider the above diagram. By Pythagoras,  $a = \sqrt{(4+x)^2 + 4}$  and  $b = \sqrt{(x-4)^2 + 16}$ ; the two roads that connects the point  $A$  to  $B$  has length  $a$  and  $b$ . The total length of the roads; that is  $a + b$  is minimised when the two lines  $AC$  and  $CB$  in the diagram are co-linear. Therefore, the combine length of the two roads is  $\sqrt{(2+4)^2 + 8^2} = 10$ .



5. (a) Since the lines  $KM$  and  $CB$  are parallel, the triangles  $\triangle KMO$  and  $\triangle OLC$  are similar. In particular, by angles and ratios we have the formula

$$\frac{|KO|}{|KL|} = \frac{|OM|}{|MC|}$$

- (b) Since  $M$  is the midpoint of  $AB$  and the line  $KM$ ,  $CB$  are parallel, by the midpoint theorem  $K$  is the midpoint of  $AL$ . Additionally, by using the fact that  $|MC| = |AL|$ , we have  $|KL| = |MC|$ . Now substituting  $|KL| = |MC|$  into the formula from part (a), we obtain  $|KO| = |OM|$ . Therefore the triangles  $\triangle KMO$  and  $\triangle OLC$  are isosceles. Finally, using the condition  $\angle OLC = 45^\circ$  we have  $\angle COL = 90^\circ$ .

6. Since the constant coefficient of  $p(x)$  is 3,  $abcd = 3$ . Therefore,

$$\frac{abc}{d} = \frac{3}{d^2}; \quad \frac{acd}{b} = \frac{3}{b^2}; \quad \frac{abd}{c} = \frac{3}{c^2}; \quad \frac{bcd}{a} = \frac{3}{a^2};$$

Let  $y = 3 - x^2$ , then  $p(\sqrt{3-y}) = 0$  when  $p(x) = 0$ . Therefore rearranging  $p(\sqrt{3-y}) = 0$  gives a polynomial of  $y$  with the required roots.