MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 12, August 2, 2015 [1]

1. (a) Show that the number $2017^{46} - 46$ is not prime.

    (b) Show that the number $2017^{46} + 46$ is not prime.

2. The polynomial $x^3 + px^2 - x + q$ has a factor $(x - 5)$ and a remainder of 24 when divided by $(x - 1)$. Find the values of p and q.

3. Let $A(-13; 14); B(-3; -5)$ and $C(7; 11)$ be the coordinate of vertices of a triangle $\triangle ABC$. Find the coordinate of the point P inside $\triangle ABC$ such that the triangle $\triangle PBC$ is equilateral.

4. An arithmetic sequence has positive integral entries. The sum of some 4 consecutive terms is 30. The some of some 5 consecutive terms is 30. Find the least number of terms overlapping in the two subsequences.

5. Let $f(x) = \frac{4^x}{4^x + 2}$. Calculate

$$ f\left(\frac{1}{2015}\right) + f\left(\frac{2}{2015}\right) + \cdots + f\left(\frac{2014}{2015}\right). $$

6. In a convex quadrilateral $ABCD$ the diagonals are perpendicular. Points M and N are marked on the sides AD and CD respectively. Suppose the $\angle ABN$ and $\angle CBM$ are right-angles, prove that the lines AC and MN are parallel.

Senior Questions

1. Let [x] denote the greatest integer less than or equal to. If α; β and γ are the roots of the function $f(x) = x^3 + x^2 - 5x - 1$, find $[\alpha] + [\beta] + [\gamma]$.

2. Show that if n and m are positive integers, then $(nm)! \ge (m!)^n (n!)^m$.

3. Find all positive numbers x and y such that

$$x^{x+y} = y^{x+2y} \quad \text{and} \quad x^{2x+y} = y^{x+4y}:$$