MATHEMATICS ENRICHMENT CLUB.

Problem Sheet 16, September 1, 2015[1]

1. Suppose we take four socks and non of them match, then the next sock we draw must form a pair with one of the socks we have taken. Now to get another pair of socks, in the worst case we have to take two more socks. Therefore we have to take at least $4 + 2 \quad 5$ socks from the draw to guarantee that ve of the pairs are matching.

2. For a number to be divisible by 12 it must also be divisible by any factors of 12; that is $2; 3; 4; 6$. Recall that for a number to be divisible by 3, then the sum of it digits must be divisible by 3. For a number to be divisible by 4, the last two digit must be divisible by 4. For a number to be divisible by 4 then it must be even. Finall, since 2 and 3 are co-prime if a number is divisible by 2 and 3 then it must be divisible by $2 \quad 3 = 6$.

   So this 5-digit number must be divisible by both 3 and 4. Since the sum of the digits of the 5-digits numbers we are forming is always 15, we need not check the divisibility by 3. Now for this 5-digits number to be divisible by by 4, then the possibility for the we can conclude that $4y$ is either zero or $4y \quad y^2 \quad 2y$

iii. $b + d \quad e$ and $a + b \quad d$. Then each of six triples $\{a; b; d\}$, $\{a; b; e\}$, $\{a; c; e\}$; $\{a; d; e\}$ $\{b; c; e\}$, $\{b; d; e\}$ does not form a triangle.
Suppose that neither of above cases takes place, that is, $b + c > d$, $c + d > e$ and at least one of inequalities $b + d > e$ and $a + b > d$ holds. We shall show that this is impossible.

iv. If $b + c > d$, $b + d > e$ then

$$a2 + b2 + c2 + d2 + e2 < ab + bc + ce + (b + c)d + (b + d)e:$$

Contradiction.

v. If $c + d > e$, $a + b > d$ then

$$a^2 + b^2 + c^2 + d^2 + e^2 < ab + bc + cd + (a + d)d + (c + d)e:$$

Contradiction.