MATHEMATICS ENRICHMENT CLUB.

Solution Sheet 9, June 23, 2015 [1]

1. Since x and y are both integers, we can only get a solution when x is an even number. If $x \neq \; y \neq 0$, then $x = \;\; 2k$ for $k = 1; 2; :::; 49$ and the corresponding values for y are $y = \;\; \frac{1}{2}(100 \;\; 2k)$; that is for each choice of $x \neq 0$, there are two choices for y. If $x = 0$ then $y = \;\; 50$. Similarly, if $y = 0$ then $x = \;\; 100$. Therefore the total number of di erent solutions is $4 \;\; 49 + 2$

4. I think there was a typo in part (a) that made it trivial once solution to (b) is founded, the question is suppose to be $x^3 - 5[x] = 10$.

   (a) x^3 must be an integer as $x^3 = 10 + 5[x]$. Also $x^3 \leq 10 + 5x$ and $x^3 > 10 + 5\lfloor x - 1 \rfloor$ so $2 < x < 3$. Hence $[x] = 2$ and therefore $x^3 = 10 + 5 \cdot 2 = 20$ and $x = \sqrt[3]{20}$.

   (b) $y^3 - 5\lfloor y \rfloor = 10$ so $10 < y^3 < 15$ and $\lfloor y \rfloor = y - 2$. Hence $y^3 = 10 + 5\lfloor y - 2 \rfloor = 5y$, and since $y \neq 0$, $y^2 = 5$ and $y = \sqrt{5}$.

5. The answer is 8.

6. Let n be an even number such that $n \geq 4$. First we prove that $5^n \equiv 625 \pmod{1000}$; that is the remainder of 5^n

To complete the question, we show that n is divisible by 5. The possible remainder of an integer a divided by 5 are $0, 1, 2, 3$ and 4, therefore any perfect square number must have remainders $0, 1^2, 2^2, 3^2$ 5 and 4^2 3(5); that is $0, 1, 4$ are the only remainders of a perfect square number when divided by 5. If we consider the remainders of $2n+1$ and $3n+1$ when divided by 5, for $n = 0, 1, 2, 3, 4$, we can see that the only time when both $2n+1$ and $3n+1$ have remainders either $0, 1, 4$ is when $n = 0$. Hence the only time when both $2n+1$ and $3n+1$ are perfect squares is when n 0 (mod 5); that is n is divisible by 5.