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MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 1, May 5, 2015

1. Let $x = 0.284284284\ldots$, then

$$\begin{aligned} 1000x &= 284.284284284\ldots \\ &= 284 + x, \end{aligned}$$

   thus $x = 284/999$.

2. We can write the finite sum as

$$1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{10100} = 1 + \sum_{n=2}^{101} \frac{1}{n \cdot (n-1)}$$

   Using the given formula,

$$\begin{aligned} 1 + \sum_{n=2}^{101} \frac{1}{n \cdot (n-1)} &= 1 + \sum_{n=2}^{101} \left(\frac{n-1}{n} - \frac{n-2}{n-1} \right) \\ &= 1 + \sum_{n=2}^{101} \frac{n-1}{n} - \sum_{n=1}^{100} \frac{n-1}{n} \\ &= 1 + \frac{100}{101}. \end{aligned}$$

3. Let S be the number of members that plays Soccer.

   (a) If we add the number of members that plays either Basketball, Cricket or Soccer, we would end up with a number that is greater than the total number of members in the sports club, because we have double counted the number of members that plays two sports *only*, and triple counted the number of members that plays *all* three.

   So to balance this out we need to subtract the double/triple counts: We know that 10 plays *all* three sports, so these member we triple counted. There is 60 members that plays two or more sports, and 10 that plays all three, therefore there is $60 - 10 = 50$ members that plays two sports *only*.

   The balanced equation is then

$$163 = S + 100 + 73 - 50 - 2(10),$$

   which gives $S = 60$.

(b) The number of members that plays both Basketball and Cricket but not Soccer is $25 - 10 = 15$, therefore $60 - 15 = 45$ members plays Soccer and Basketball or Soccer and Cricket or all three sports. Since $S = 60$, $60 - 45 = 15$ of these members plays Soccer only.

4. (a) Here $|QC|$ means the length of QC. By construction, the length of AP is b; that is $|AP| = b$. Since the point Q is the intersection of the tangent PQ and CQ of the same circle arc PC, $|PQ| = |CQ|$ (you may want to prove this as an exercise). So the problem is reduced to finding $|PQ|$. Note that BPQ