MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 16, September 5, 2016

1. Let n be a positive integer. If the polynomial

$$(x + 1)(x + 2)(x + 3)\cdots(x + n)$$

is expanded, find the sum of the coefficients of odd powers of x.

2. Four points are located in a plane. For each point, the sum of the distances to the other three is calculated; and these four sums are found to be the same. Determine all

Senior Questions

1. Unit cubes are arranged into an 20 18 15 block. A straight line is drawn from one corner of the block to the diagonally opposite corner. How many unit cubes does the line pass through?

2. A collection of 2016 numbers consists of one zero and 2015 ones.

   (a) It is permitted to choose any two numbers from the collection and replace each of them by the average of the two. Is it possible by repeating this operation to obtain a collection in which all 1995 numbers are the same?

   (b) It is permitted to choose any two or more of the numbers (but not the whole collection) and replace each of them by the average of the chosen numbers. Is it possible to make all the numbers equal?

3. The graphs of four functions of the form $y = x^2 + ax + b$, where a and b are real coe cients, are plotted on the coordinate plane. These graphs have exactly four points of intersection, and at each one of them, exactly two graphs intersect. Prove that the sum of the leargest and the smallest x-coordinate of the points of intersection is equal to the sum of the other two.