[redacted]

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 17, September 12, 2016

1. Find all positive integer solution pairs (x, y) for the equation
$$x^2 + xy - y - 1 = 2016.$$

2. There are ten cards with the number a on each, ten with b and ten with c, where a, b and c are distinct real numbers. For every five cards, it is possible to add another five cards so that the sum of the numbers on these ten cards is 0. Prove that one of a, b and c is 0.

3. One day, Albert from Anastasia and Associates Attorneys and Betty from Bartholomew and Brothers Barristers leave at the same time to deliver messages to the other legal firm. They follow the same route, walking at constant but different speeds and pass each other when Albert has walked A metres. After delivering their messages, and each waiting 10 minutes for a reply, they return to their own firm at the same speeds

Senior Questions

1. Let a and b be arbitrary positive integers. The sequence $\{x_k\}$ is defined by $x_1 = a$, $x_2 = b$ and for $k \geq 3$, x_k is the greatest common divisor of $x_{k-1} + x_{k-2}$.

   (a) Prove that the sequence is eventually constant.

   (b) How can this constant value be determined from a and b?

2. In an arbitrary binary number, consider any digit 1 and any digit 0 which follows it, not necessarily immediately. They form an odd pair if the number of other digits in between is odd, and an even pair if this number is even. Prove that the number of even pairs is greater than or equal to the number of odd pairs.

3. There are $n \geq 4$ points on a circle, numbered 1 to n in some order. Two non-neighbouring points A and B are said to be "linked" if points on at least one of the two arcs between A and B all have numbers smaller than those of both A and B.

   Prove that the number of "linked" pairs of points is exactly $n - 3$.