



Senior Questions

1. A sequence is defined as follows: $a_1 = a_2 = a_3$, and for all positive integers n , $a_{n+3} = a_{n+2} + a_{n+1} + a_n$. Given that $a_{28} = 60903017$, $a_{29} = 11201821$ and $a_{30} = 20603361$. Find the last 3-digits of $\sum_{k=1}^{28} a_k$.
2. Given that x is the last 2 digits of the sum:

$$\sum_{n=0}^{x^{00}} n!$$

Find the last 2 digits of

$$\sum_{m=0}^{x^{00}} m^x$$

3. On an island there are a number of towns, and a number of roads linking the towns. Each town is the junction of exactly three roads. A traveler sets out along a road from one town, and at the next town takes the left hand road of the two available. At the following town the right hand road is taken, and so on, with left and right turns alternating. Prove that at some stage the traveler must return to the first town.