Senior Questions

1. Given that $n > 1$ is prime if and only if the remainder of $(n - 1)!$ is $n - 1$ divided by n. Solve the system of simultaneous equations:

$$p! + 1 = (2p + 1)^2$$
$$q! + 1 = (10q + p - 4)^2 \ .$$

2. Solve

$$\int_{-\frac{p_2}{2}}^{1} \frac{1}{x^5 \sqrt[p]{4x^2 - 1}} \, dx \text{:}$$

3. Let C be a right angle in triangle ABC. On legs AC and BC the square $ACKL$, $BCMN$ are constructed outside of the triangle. If CE is an altitude of the triangle; see diagram below. Prove that LEM is right angle.