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MATHEMATICS ENRICHMENT CLUB.

Solution Sheet 2, May 7, 2016

1. If the first digit of n is $1, 2$ or 9, then there is nothing to prove. If the first digit of n is 3, then the first digit of $3n$ is 9. If the first digit of n is $4, 5$ or 6, then the first digit of $3n$ is 1. If the first digit of n is 7 or 8, then the first digit of $3n$ is 2. This completes the proof, has we have exhausted all possibilities.

2. How many numbers between 100 and 500 that are divisible by 7 but not by 21.

   The first and last numbers divisible by 7 between 100 and 500 are 105 and 497 respectively. Therefore, there are $(497 - 105)$

Therefore,

$$\int_0^1 (f(x))^2 dx \quad 2\ \ _1 + 2\ \ _2 \qquad \ ^2_1 + \ \ _1\ \ _2 + \ \ ^2_2{=}3 \ : \qquad (1)$$

In particular, the minimal of $\int_0^1 (f(x))^2 dx$ is attained when the RHS of (1) is minimal. Let $F(\ _1;\ \ _2) = 2\ \ _1 + 2\ \ _2 \quad (\ ^2_1 + \ \ _1\ \ _2 + \ \ ^2_2{=}3)$. To nd the critical points of F, we solve the system

$$\frac{@F}{@\ \ _1} = \quad 2 + 2\ \ _2 + \ \ _2 = 0; \qquad , \text{w}\ \ _2 + 2$$

3. If $a = 0$, then $b = 2$ is the only solution.

   Now if $a > 0$, then $3 \quad 2^a$ is always even, so that b is always odd. Hence $b = 2k + 1$ for some integer k. Hence, $3 \quad 2^a = 4k^2 + 4k = 4(k + 1)k$.

   If $a = 1$, then $3 = 2(k + 1)k$, which has no solutions.

   If $a = 2$, then $3 = (k + 1)k$, again has no solutions.

   If $a > 2$, then $3 \quad 2^{a \ 2} = (k + 1)k$. Since 2 and 3 are coprime, if k is odd, then $k + 1 = 2^{a \ 2}$ and $k = 3$. Else if k is even, then $k + 1 = 3$ and $k = 2^{a \ 2}$. Hence, we