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MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 8, June 20, 2016

1. There are ${}^{12}C_3 = \frac{12\ \ 11\ \ 10}{3\ \ 2} = 220$ ways to choose 3 Tim-Tams randomly from the 12 Tim-Tams in the box. Therefore, the chance for one person to choose 3 white Tim-Tams is 1 out of the 220. Hence, for exactly one of four person to end up with 3 white Tim-Tams is $4 \quad \frac{1}{220} = \frac{1}{55}$.

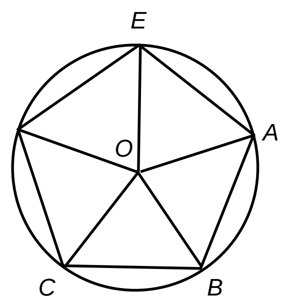
2. It is possible to get it as small as the integer 2. Here is one way to play the game to get to 2. First take 2014 and 2016, then we delete them from the board and add in 2015. Next, we take the two 2015 (one from the start and one obtained from the previous step), then we delete them from the board and add on a single 2015. From this point and onwards, we can delete the largest two integers, which is always in the form $n$ and $n \quad 2$, for some integer $n < 2016$, and add in their average $(n \quad 1)$. In this way, we will eventually get to: $5; 3; 2; 1$, then $4; 4; 2; 1$, then $3; 2; 1$, then $2; 2$ then 2.

3. (a) We can employ the divisibility by 9 rule to solve this. Recall that an integer is divisible by 9 if and only if the sum of its digits is divisible by 9. The numbers $18; 108; 1008; \ldots$ all have sum of digits equal to 9. Thus each $18; 108; 1008; \ldots$ is divisible by 9. Moreover, each $18; 108; 1008; \ldots$ is an even integer, and thus is divisible by 2. Therefore, every term in the sequence $18; 108; 1008; \ldots$ is divisible by 18.

   (b) We can also solve this equation with mathematical induction. One can represent the terms of the in nite sequence as $a_n = 10^n + 8$. Then clearly $a_1$

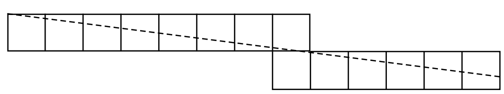
of $2; 3; 5; 7; 11$ or 13. Therefore, to solve this problem one only needs to count the number of factors of $2^3; 3^3; 5^3; 7^3; 11^3; 13^3$ the number 40! contains. The nal solution is $(12 + 1)\quad(6 + 1)\quad(3 + 1)\quad(1 + 1)\quad(1 + 1)\quad(1 + 1) = 2912$.

5. (a) See diagram below. Let $\backslash OAB =$ and $\backslash OAE =$ . Then each of the angles of the pentagon is $+$ . Since $OA = OB$, $4OAB$ is isosceles. Therefore, $\backslash OBA =$ . Moreover, since $\backslash ABC = \ +$ and $\backslash OBA =$ , one has $\backslash OBC =$ . Similarly, $\backslash OCD = \backslash ODC =$ and eventually we get $\backslash OEA = \backslash OAE =$ . Hence $=$ . Therefore all ve triangles in the gure are congruent, and the ves sides of the pentagon are all equal.




(b)

6. We can create mirror images of the initial square billiard board, so that the ball travels in a straight line; as shown below. Then to solve this problem, one counts the number



of copies of the billiard is required for the ball to travel from one corner to another. Since 19 and 96 are co-prime, the number of copies required is $19 + 96 \quad 1 = 114$.

3. For any odd integer ***n***,,