MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 12[1], August 14, 2017

1. Let m and n be positive numbers satisfying:
   (a) The sums of its divisors coincide.
   (b) The sums of the reciprocal of its divisor also coincide.

5. There are many examples of twin primes: pairs $(p; p + 2)$ in which both are prime numbers (for example $(3; 5)$, $(11; 13)$, $(41; 43)$,...). In fact, mathematicians believe that there are infinitely many twin primes but we still do not know how to prove this.

   We will look at a much simpler case: let us define prime triplets as 3 consecutive odd primes $(p; p + 2; p + 4)$. Are there any besides $(3; 5; 7)$? If not, how do you prove it?

Senior Questions

We will devote these questions to understand a bit more in detail the Harmonic series:

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

1. Show that the Harmonic series is greater than any given number $M > 0$. Then show that if we only take the sum of reciprocals of even numbers, the sum is also greater than any given number.

2. If $n > 1$. Prove that the partial sum

$$H_n = \sum_{k=1}^{n} \frac{1}{k}$$

is never an integer.

3. Consider the sum of reciprocals of all natural numbers that do not contain the number nine (when written in decimal expansion)

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \dots$$

(we avoid $1/9, 1/19, 1/29, \dots$ and so on).

Show that in fact the sum of the remainder terms is smaller than 10.