

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 16¹, September 11, 2017

1. Let $N = 11!$. How many positive cubes divide N ?
2. Suppose that $\log(x^2y^{2^n}) = 1$ and $\log(x^{2^n}y^2) = 1$ where x and y are positive numbers and n is a positive integer. Show that

$$\log(x^n y^n) < 1:$$

3. Simplify

$$\frac{1}{\frac{1}{2} + \frac{1}{1}} + \frac{1}{\frac{1}{3} + \frac{1}{2}} \frac{1}{\frac{1}{4} + \frac{1}{3}} + \dots + \frac{1}{\frac{1}{3457} + \frac{1}{3456}}:$$

4. In a square $ABCD$, E is a point on diagonal BD . P and Q are the circumcentres of $\triangle ABE$ and $\triangle ADE$ respectively. Prove that $APEQ$ is a square.



5. Let n be a positive integer and suppose that the numbers 2^n and 5^n begin with the same digit. Prove that there is only one possibility for this digit.
6. On a $4 \times n$ chessboard we wish to place $2n$ knights in such a way that none attacks any other.

¹Some of the problems were published in Parabola.

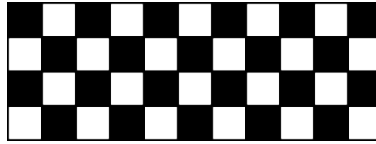
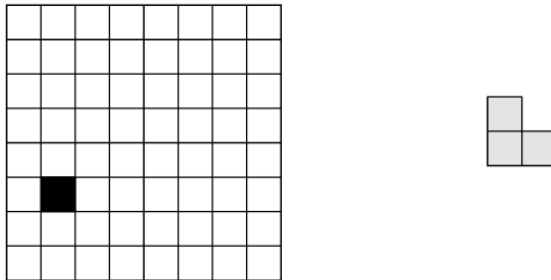


Figure 1: Example of a $4 \times n$ chessboard for $n = 11$.

- (a) Give three possible ways of doing this.
- (b) Prove that there is no closed knights tour on the $4 \times n$ board (one knight that moves along the chessboard passing only once on each position and returning to the starting point).

Senior Questions

1. You are in a really strange car riding away from your house. This particular car cannot turn, nor can you control the speed. In fact, the speed is always exactly equal to your distance from your house. For example, when you are two km (or whatever unit) away, you will be travelling two km per hour. At 2.5 km away, you will be travelling 2.5 km per hour. Assuming you start one km away from your house, how far will you be after one hour?
2. Find the area bounded by $y = \log x$ and $y = (\log x)^2$.
3. Consider a $2^n \times 2^n$ chessboard with one (arbitrarily chosen) square removed, as in the following picture (for $n = 3$):



Prove that any such chessboard can be tiled without gaps or overlaps by L-shapes consisting of 3 squares each.