MATHEMATICS ENRICHMENT CLUB.

Solution Sheet 10, July 31, 2017

1. There are 49 ways, and even more methods of arriving at this answer. Perhaps the easiest is to use cases starting with using 0; 1 or 2 possible 50 cent coins.

2. Divide the grid into nine 1

4. (a) $29 = 5^2 + 2^2$, $37 = 6^2 + 1^2$. For 30, note that none of the following are square numbers:

$$301 = 29;\ 304 = 26;\ 309 = 21;\ 3016 = 14;\ 3025 = 5:$$

Similarly, 31 cannot be expressed as a sum of two squares.

(b) Easy.

(c) $1073 = (5^2 + 2^2)(6^2 + 1^2) = (302)^2 + (5 + 12)^2$.
Swapping $5^2 + 2^2$ with $2^2 + 5^2$ yields $1073 = 7^2 + 32^2$.

5. Pigeon-hole principle. Each number can be written in the form $2^k(2m + 1)$ where $k; m \quad 0$. Since each number is less than 1001, m must be less than 500.

So since you're choosing 501 numbers, two of the numbers must have the same m value.

These two numbers can be written as $2^{k_1}(2m + 1)$ and $2^{k_2}(2m + 1)$.

Either $k_1 \quad k_2$ or $k_2 \quad k_1$, so without loss of generality, assume $k_1 \quad k_2$. Then $2^{k_1}(2m + 1)$ divides $2^{k_2}(2m + 1)$, which concludes the proof.

Senior Questions

The number of ways to obtain k when rolling two dices coincides with the coe cient of $x^k$ in

$$f(x) = (x + x^2 + x^3 + x^4 + x^5 + x^6)^2:$$

Now, note that

$$f(x) = x^{12} + 2x^{11} + 3x^{10} + 4x^9 + 5x^8 + 6x^7 + 5x^6$$

xxT6e2