

MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 13, August 21, 2017

1. Firstly, we factorise the left hand side of the equation.

$$3x^2 - 8xy + 4y^2 = (3x - 2y)(x - 2y)$$

and so on. There are four solutions in total: (4;5), (- 4; 5), (4;3) and (- 4; 3).

2. Complete the square, then take difference of two squares. Answers are $(x^2 - 2x + 2)(x^2 + 2x + 2)$ and $(x^2$

$$- 2x + 1)(x^2 + 2x + 1).$$

3. Suppose $x = y = z$. Then $5=8 = 1=x + 1=y + 1=z - 3=x$, fo $x < 5$. This means there are only 4 possible values for x .

$x = 1$: No solution

$x = 2$: Solve $1=y + 1=z = 1=8$. So $8 - y = 2 = 8$. Testing y values in this range gives (9;72) and (10;40)

$x = 3$: Solve $1=y + 1=z = 7=24$. Since $1=4 < 1=y + 1=z < 1=3$. So $3 - y = 2 = 4$. Testing y values in this range gives (4;24) and (6;8)

$x = 4$: Solve $1=y + 1=z = 3=8$. Answers (3;24) and (4;8)

4. The octagon is not regular!

5. Treat it as an arithmetic progression. Answer is 4.

6. (a) $(7) = 2$, $(10) = 4$ and $(25) = 3$.

(b) If $\tau(m) = 2$, m is a prime; if $\tau(m)$ is odd, then m is a square.

(c) Use the prime factorisation of n . If $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$ for distinct primes, p_1, \dots, p_k , then $\tau(n) = (a_1 + 1)(a_2 + 1) \cdots (a_k + 1)$.

Senior Questions

1. Use partial fractions to express S as a telescoping sum. Thus $\frac{1}{(3n-2)(3n+1)} =$