MATHEMATICS ENRICHMENT CLUB.

Solutions to Problem Sheet 16, September 11, 2017

1. We have

$$\begin{aligned} N! &= 11 \quad 10 \quad 9 \quad 8 \quad 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \\ &= 11 \quad 7 \quad 5^2 \quad 3^4 \quad 2^8 \\ &= (11 \quad 7 \quad 5^2 \quad 3 \quad 2^2) \quad (3 \quad 2^2)^3 \end{aligned}$$

4. First we know that $\angle APE = 2\angle ABE = 90^o$ and $\angle AQE = 2\angle ADE = 90^o$. Also, $\triangle APE$ is isosceles as $AP = PE$.

Senior Questions

1. Note that you're travelling on a straight line (i.e., distance=displacement), since your car cannot turn.

   In that case, we can say that the position, given as a function $f(t)$ of time, has a derivative (instantaneous velocity) which is equivalent to $f(t)$, which is the distance travelled from the starting position (the house). This means that $f'(t) = f(t)$.

   It is well known that, in this case, $f(t) = Ce^t$ for some constant C. Taking into account that $f(0) = 1$ (we start at 1km from our house). Therefore, after one hour, we are e kms away.

2. The points of intersection are $A = (1; 0)$ and $B = (e; 1)$. In the region we are interested, $1 \quad x \quad e$, we have $0 < \log x < 1$ hence $0 < \log^2 x < \log x$. Which means that the graph of the function $\log x$ lies above the graph of $\log^2 x$.

   The area we are looking for can be expressed as:

$$\int_1^e (\ln x \quad \ln^2 x)dx:$$

   We have

$$\int_1^e \log x dx = [x \log x \quad x]_1^e = (1 \quad e \quad e) \quad (0 \quad 1) = 1$$

   and

$$\int_1^e \log^2 x dx = \quad x \ln^2 x \quad 2x \ln x + 2x \,{}_1^e = e \quad 2:$$

   Hence the area equals 3 eur, we e-32227(ar05)5is1tlhe rlofeyThsn this rl(p)-27ertay. The casy cres
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