

MATHEMATICS ENRICHMENT CLUB.  
Problem Sheet 6, June 11, 2018

1. Solve  $x$

## Senior Questions

1. Prove that the square of the  $n$ th triangular number is the sum of the first  $n$  cubes, i.e.

$$\sum_{k=1}^n k^2 = \sum_{k=1}^n k^3$$

2. Find the limit  $\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$ .

3. Let  $f(x) = xe^x$ .

(a) Draw the graph of  $y = f(x)$ , clearly indicating any stationary points on your diagram.

(b) For  $x > e^{-1}$ ,  $f(x)$  has an inverse,  $f^{-1}(x)$ . Add the graph of  $y = f^{-1}(x)$  to your diagram.

(c) This inverse is the principal branch of the Lambert  $W$  function, and is also known as the Omega function or the product-log function. We will denote it by  $W(x)$ . Show that

$$\frac{dW}{dx} = \frac{W(x)}{x(1 + W(x))}$$