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MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 10, 6 August, 2018

1. Let S be the number of members that play Soccer.

   (a) If we add the number of members that play either Basketball, Cricket or Soccer, we end up with a number that is greater than the total number of members in the sports club, because we have double counted the number of members that plays two sports *only*, and triple counted the number of members that plays *all* three.

   So to balance this out we need to subtract the double/triple counts: We know that 10 members play *all* three sports, so these members we triple counted. There are 60 members that plays two or more sports, and 10 that plays all three, therefore there are $60 - 10 = 50$ members that plays two sports *only*.

   The balanced equation is then

   $$163 = S + 100 + 73 - 50 - 2(10);$$

   which gives $S = 60$.

   (b) The number of members that play both Basketball and Cricket but not Soccer is $25 - 10 = 15$, therefore $60 - 15 = 45$ members plays Soccer and Basketball or Soccer and Cricket or all three sports. Since $S = 60$, $60 - 45 = 15$ of these members play Soccer only.

2. (a) Let a_1, k $a_k + 10^{k-1}a_{k-1} + \dots + 10^2 a_3 + 10a_2 + a_1.$

Since 10^i is divisible by 4 for $i = 2, 3, \dots, k$, if n is divisible by 4, then so is $10a_2 + a_1$, which is the number formed by the last two digits of n.

(b) Let m be the number formed by the sum of all of the digits of n; that is

$$m = a_k + a_{k-1} + \dots + a_2 + a_1.$$

Consider the difference

$$n - m = (10^k - 1)a_k + (10^{k-1})a_{k-1} + \dots + 99a_3 + 9a_2.$$

Clearly $n - m$ is a multiple of 9, so if n is divisible by 9, then so is m.

3. We can write the finite sum as

$$1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{10100} = 1 + \sum_{n=2}^{101} \frac{1}{n(n-1)}$$

Using the given formula,

$$\begin{aligned} 1 + \sum_{n=2}^{101} \frac{1}{n(n-1)} &= 1 + \sum_{n=2}^{101} \left(\frac{n-1}{n} - \frac{n-2}{n-1} \right) \\ &= 1 + \sum_{n=2}^{101} \frac{n-1}{n} - \sum_{n=1}^{100} \frac{n-1}{n} \\ &= 1 + \frac{100}{101}. \end{aligned}$$

4. Let the number we wish to express as a continued fraction be n. As given in the hint, $a_0 = \lfloor n \rfloor$, which is easy to calculate. Then

$$n - a_0 = \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \dots}}};$$

and taking reciprocals, we have

$$\frac{1}{n - a_0} = a_1 + \frac{1}{1 + \frac{1}{a_2 + \frac{1}{a_3 + \dots}}}.$$

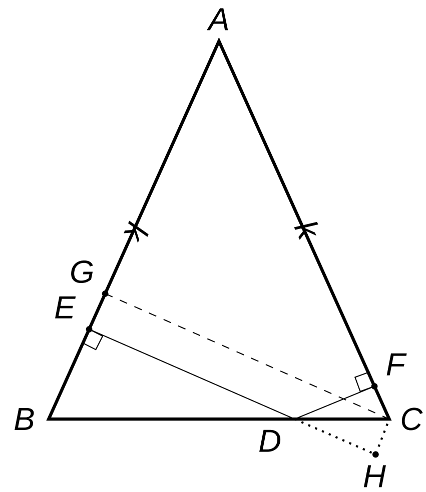
Once again, we can see that a_1 is the integer part of $\frac{1}{n - a_0}$. By iteratively taking reciprocals and finding integer parts, we can determine the values of $a_1, a_2, a_3, \dots$. You should obtain the following results:

(a) $\frac{355}{113} = [3; 7, 16]$

(b) $\frac{113}{355} = [0; 3, 7, 16]$

(c) $\sqrt{2} = [1; 2, 2, 2, \dots]$

(d) $\frac{1}{\sqrt{2101}}$



Now \ABC = \ACB, since 4ABC is isosceles, and since 4EBD and