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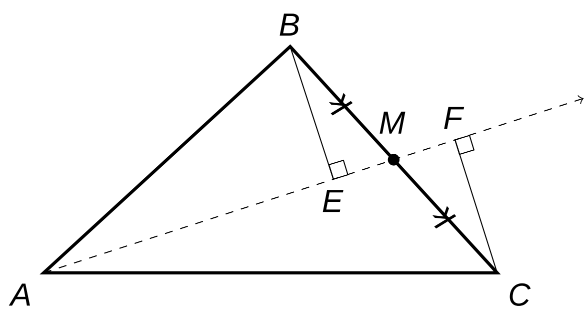
MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 11, 13 August, 2018

1. Let BE and CF be perpendiculars dropped from B and C to AM, extended if necessary. We need to prove that $BE = CF$.



Since BE and CF are both perpendicular to AM, $\backslash BED = \backslash DFC = 90$, and since AM is a median, $BM = CM$. Moreover, $\backslash$

4. (a) $0.75_{10} = 0.11_2$, since $0.75 = \frac{1}{2} + \frac{1}{4} = 1 \quad \frac{1}{2^1} + 1 \quad \frac{1}{2^2}$

(b) $0.96875_{10} = 0.11111_2$ in base 2.

(c)

$$\frac{1}{2} + \frac{1}{4} + \quad \frac{1}{2^k} + \quad = 0.1_2 = 1$$

If you are not convinced of this last fact, let $x = 0.1_2$. Then

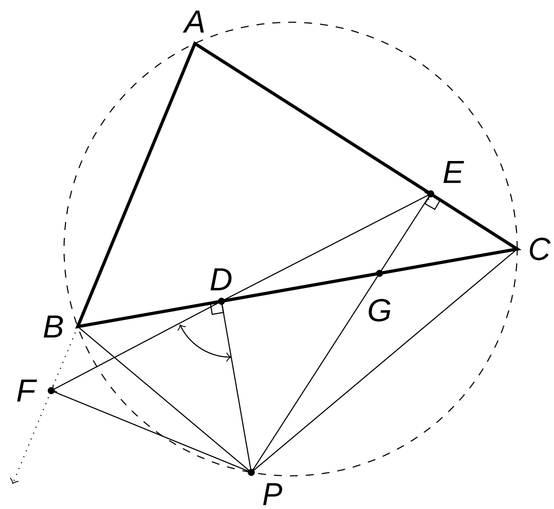
$$\begin{aligned} 2x &= 1.1_2 && (1) \\ x &= 0.1_2 && (2) \\ \hline x &= 1 && (1) \quad (2) \end{aligned}$$

5. Firstly, we note that $x^3 \quad y^3 = (x \quad y)(x^2 + xy + y^2)$. Then, we nd the prime factorisation of $1729 = 7 \quad 13 \quad 19$. Thus the possible factors of 1729 are 1, 7, 13, 19, 91, 133, 247, and 1729 itself. If we assume that $x \quad y = 1$, then

$$x^2 + xy + y^2 = 1729.$$

Furthermore, we can substitute $x = y + 1$ into this second equation, thereby obtaining a quadratic in y. In this case, the quadratic does not have integer solutions, as is not a perfect square. However, continuing this way through all the possibilities, we obtain the solutions $(\quad 1; 12)$, $(1; \quad 12)$, (

2. Join CP and PB as shown.



Let $\backslash ACB =$, $\backslash BCP =$ and $\backslash FDP =$. Let D and E be the feet of perpendiculars from P