[redacted]

MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 9, July 30, 2018

1. The angles in the triangle are, in ascending order 2 , 3 , 4 for some value of . By the angle sum of the triangle,

$$
\begin{aligned}
2 \;\; + 3 \;\; + 4 \;\; &= 180^{c} irc \\
9 \;\; &= 180 \\
) \;\; &= 20
\end{aligned}
$$

   Thus the largest angle is 80 .

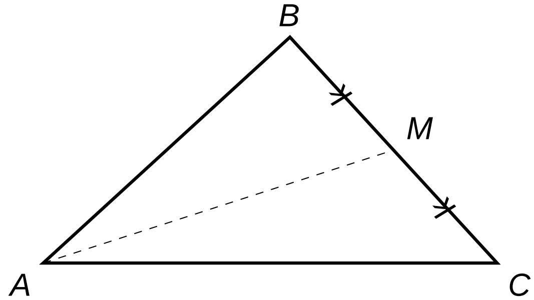
2. You can work this out on your calculator using the $\log_{10}$ button.

$$
\begin{aligned}
\log_{10}(125)^{100} &= 100 \log_{10}(125) \\
&= 209{:}69 : : :
\end{aligned}
$$

   Now we can tell the number of digits of a number n by considering the integer part of $\log_{10}(n)$. If $b\log_{10}(n)c = k$, then n has $k + 1$ digits, so we can see that 100^{125} has 210 digits.

3. Applying the triangle inequality to $4AMB$, we have

$$
\begin{aligned}
AM &< AB + BM \\
) \; AM &< AB + \frac{1}{2}BC{:}
\end{aligned}
$$



Similarly, applying the triangle inequality to $4AMC$, we have

$$AM < AC + \frac{1}{2}BC:$$

If we add these two inequalities, we have

$$2AM < AB + BC + AC:$$

Thus

$$AM < \frac{1}{2}(AB + BC + AC):$$

4. We can write   as

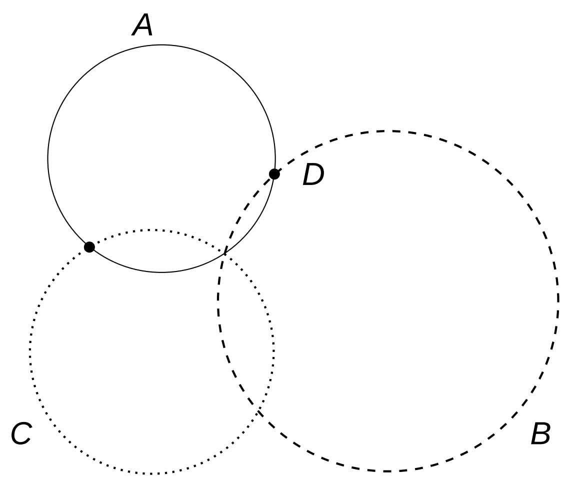
$$\begin{aligned} &= \frac{1}{1 + \quad} \\ (1 + \quad) &= 1 \\ {}^2 + \quad\quad 1 &= 0 \end{aligned}$$

This is just a quadratic in , so

$$\begin{aligned} &= \frac{1 \quad \sqrt{1 \quad 4(\quad 1)}}{2} \\ &= \frac{1 \quad \sqrt{5}}{\quad} \end{aligned}$$

Senior Questions

1. We do this by letting the circles ADE and BDF intersect at a point G. We will then prove that $ECFG$ is a cyclic quadrilateral.



and solve simultaneously to obtain $A = \dfrac{(n+m)}{2}$ and $B = \dfrac{(n-m)}{2}$.
Consequently,

$$\begin{aligned}\frac{(n+m)}{2} &= \frac{(4k-1)}{4}\\ &= \frac{(4k-1)}{2(n+m)}; \quad \text{if } n \neq -m.\end{aligned}$$

Or

$$\begin{aligned}\frac{(n-m)}{2} &= \frac{(4k+1)}{4}\\ &= \frac{(4k+1)}{2(n-m)}; \quad \text{if } n \neq m.\end{aligned}$$